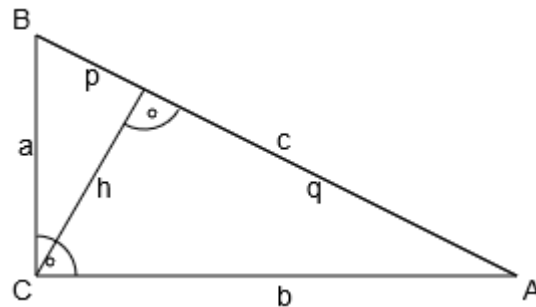


# Mathematik > Geometrie > Satz des Pythagoras

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## Satz des Pythagoras

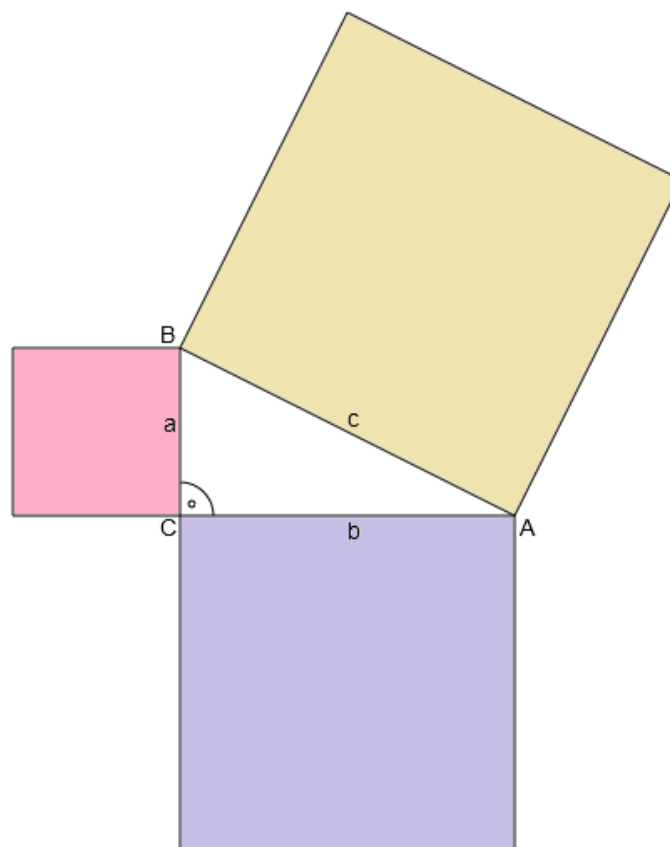
Gegeben sei ein rechtwinkliges Dreieck  $\triangle ABC$  mit den Ecken A, B, C, den (den Ecken jeweils gegenüberliegenden) Seiten a, b, c und einem rechten Winkel an der Ecke C ( $\gamma = 90^\circ$ ). Die gegenüber dem rechten Winkel liegende Seite c heißt Hypotenuse, die an dem rechten Winkel liegenden Seiten a und b Katheten.



Rechtwinkliges Dreieck

Im rechtwinkligen Dreieck gilt dann der sog. Satz des Pythagoras, wonach die Summe der Quadrate der Kathetenlängen das Quadrat der Hypotenusenlänge ergibt:

$$a^2 + b^2 = c^2.$$



Satz des Pythagoras

Durch Umstellen der Formel gilt noch:

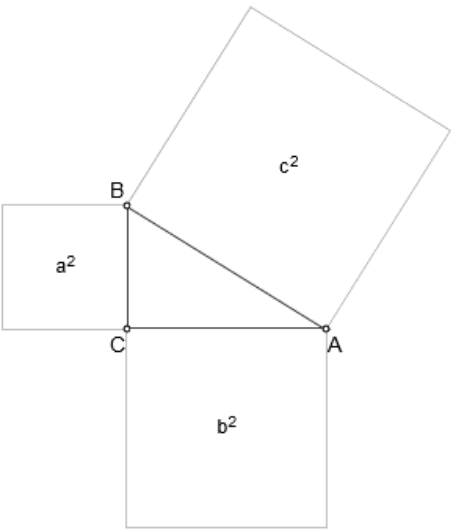
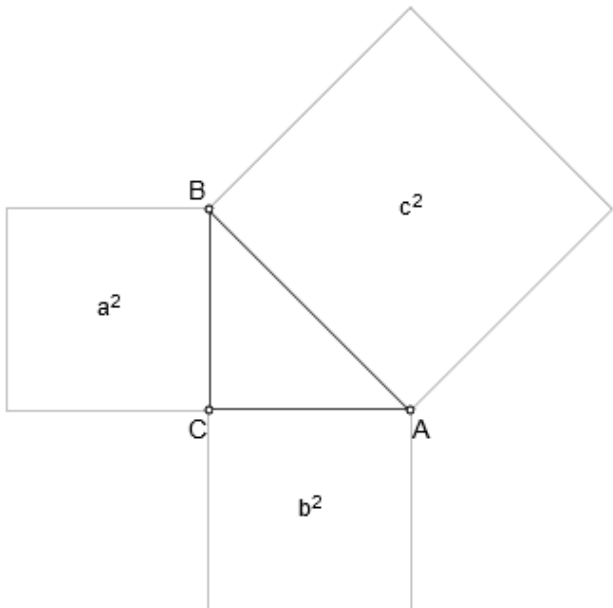
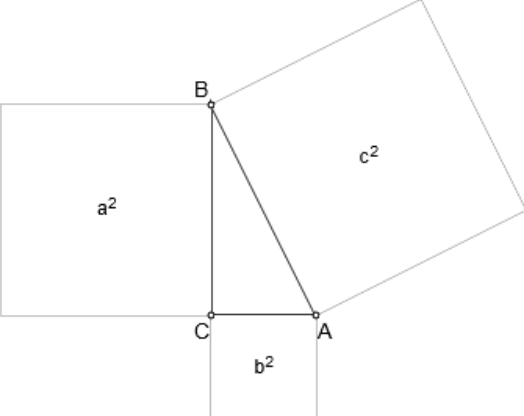
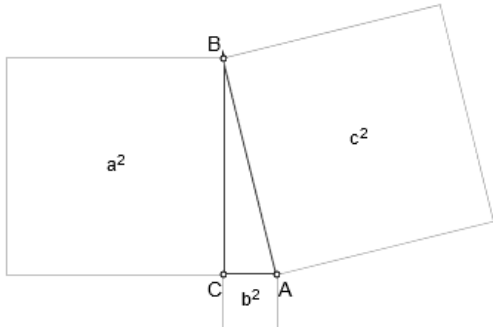
$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2} \text{ (Hypotenuse)}$$

$$a^2 = c^2 - b^2 \Rightarrow a = \sqrt{c^2 - b^2} \text{ (Kathete)}$$

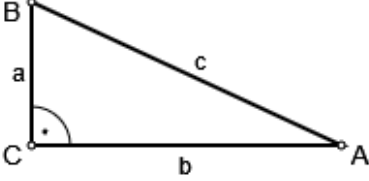
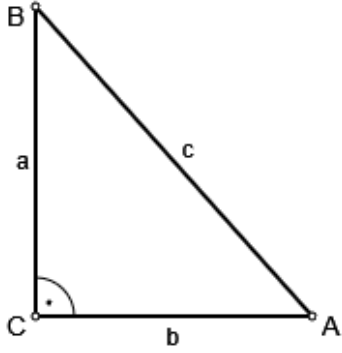
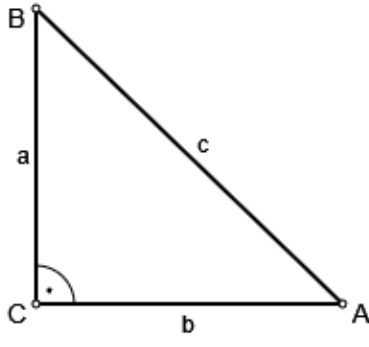
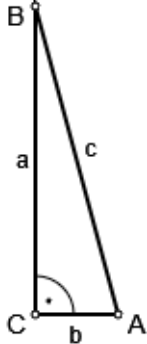
$$b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2} \text{ (Kathete)}$$

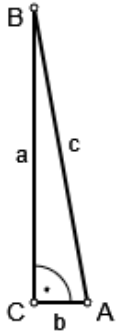
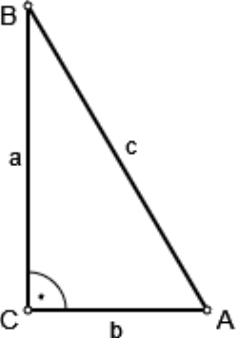
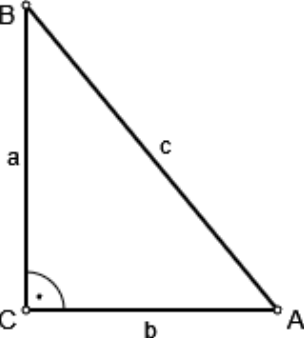
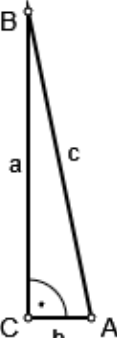
Im Übrigen gilt die Umkehrung des Satzes des Pythagoras: Verhalten sich die drei Seiten  $a$ ,  $b$ ,  $c$  in einem Dreieck  $\triangle ABC$  gemäß  $a^2 + b^2 = c^2$ , so ist das Dreieck rechtwinklig mit einem rechten Winkel an der Ecke  $C$ .

Beispiele (rechtwinklige Dreiecke):

<p>a)  <math>a = 3 \text{ cm}</math>, <math>b = 4 \text{ cm}</math>, <math>c = 5 \text{ cm}</math>  <math>a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25 = 5^2 = c^2</math></p> 	<p>b)  <math>a = 4 \text{ cm}</math>, <math>b = 4 \text{ cm}</math>, <math>c = 4\sqrt{2} \text{ cm}</math>  <math>a^2 + b^2 = 4^2 + 4^2 = 16 + 16 = 32 = (4\sqrt{2})^2 = c^2</math></p> 
<p>c)  <math>a = 4 \text{ cm}</math>, <math>b = 2 \text{ cm}</math>, <math>c = 2\sqrt{5} \text{ cm}</math>  <math>a^2 + b^2 = 4^2 + 2^2 = 16 + 4 = 20 = (2\sqrt{5})^2 = c^2</math></p> 	<p>d)  <math>a = 4 \text{ cm}</math>, <math>b = 1 \text{ cm}</math>, <math>c = \sqrt{17} \text{ cm}</math>  <math>a^2 + b^2 = 4^2 + 1^2 = 16 + 1 = 17 = (\sqrt{17})^2 = c^2</math></p> 

**Aufgabe 1:** Berechne die fehlende Seitenlänge im rechtwinkligen Dreieck  $\Delta ABC$  (Winkel  $\gamma = 90^\circ$ ,  $a$ ,  $b$  = Katheten,  $c$  = Hypotenuse).

Nr.	Gegeben:	Grafik:
1	$a = 1.2 \text{ cm}$ , $c = 2.9 \text{ cm}$	
2	$a = 9.4 \text{ cm}$ , $c = 12.6 \text{ cm}$	
3	$b = 2.5 \text{ cm}$ , $c = 3.5 \text{ cm}$	
4	$a = 7.8 \text{ cm}$ , $c = 8.1 \text{ cm}$	

5	$a = 6 \text{ cm}, b = 1.1 \text{ cm}$	
6	$a = 5.8 \text{ cm}, c = 6.7 \text{ cm}$	
7	$b = 5.1 \text{ cm}, c = 8 \text{ cm}$	
8	$a = 9.8 \text{ cm}, b = 2 \text{ cm}$	

Lösungen:

Nr.	Gegeben:	Lösungen:
1	$a = 1.2 \text{ cm}, c = 2.9 \text{ cm}$	$a = 1.2 \text{ cm}, b = 2.6 \text{ cm}, c = 2.9 \text{ cm}$
2	$a = 9.4 \text{ cm}, c = 12.6 \text{ cm}$	$a = 9.4 \text{ cm}, b = 8.4 \text{ cm}, c = 12.6 \text{ cm}$
3	$b = 2.5 \text{ cm}, c = 3.5 \text{ cm}$	$a = 2.4 \text{ cm}, b = 2.5 \text{ cm}, c = 3.5 \text{ cm}$
4	$a = 7.8 \text{ cm}, c = 8.1 \text{ cm}$	$a = 7.8 \text{ cm}, b = 2.1 \text{ cm}, c = 8.1 \text{ cm}$
5	$a = 6 \text{ cm}, b = 1.1 \text{ cm}$	$a = 6 \text{ cm}, b = 1.1 \text{ cm}, c = 6.1 \text{ cm}$
6	$a = 5.8 \text{ cm}, c = 6.7 \text{ cm}$	$a = 5.8 \text{ cm}, b = 3.4 \text{ cm}, c = 6.7 \text{ cm}$
7	$b = 5.1 \text{ cm}, c = 8 \text{ cm}$	$a = 6.2 \text{ cm}, b = 5.1 \text{ cm}, c = 8 \text{ cm}$
8	$a = 9.8 \text{ cm}, b = 2 \text{ cm}$	$a = 9.8 \text{ cm}, b = 2 \text{ cm}, c = 10 \text{ cm}$

**Aufgabe 2:** Berechne die fehlende Seitenlänge im rechtwinkligen Dreieck  $\triangle ABC$  (Winkel  $\gamma = 90^\circ$ ,  $a$ ,  $b$  = Katheten,  $c$  = Hypotenuse).

Nr.	Gegeben:
1	$a = 7.4 \text{ mm}$ , $c = 16.1 \text{ mm}$
2	$a = 13.8 \text{ dm}$ , $c = 19.9 \text{ dm}$
3	$b = 10.9 \text{ mm}$ , $c = 16 \text{ mm}$
4	$a = 23.7 \text{ dm}$ , $c = 38.3 \text{ dm}$
5	$b = 21.7 \text{ m}$ , $c = 22.5 \text{ m}$
6	$a = 22.4 \text{ mm}$ , $b = 12.7 \text{ mm}$
7	$a = 23.7 \text{ cm}$ , $c = 46.3 \text{ cm}$
8	$a = 36.6 \text{ m}$ , $b = 19.7 \text{ m}$
9	$a = 23.9 \text{ dm}$ , $b = 33.4 \text{ dm}$
10	$a = 32.8 \text{ dm}$ , $b = 21.7 \text{ dm}$
11	$b = 10.7 \text{ dm}$ , $c = 35.3 \text{ dm}$
12	$a = 25.6 \text{ mm}$ , $c = 38.4 \text{ mm}$

Lösungen:

Nr.	Gegeben:	Lösungen:
1	$a = 7.4 \text{ mm}$ , $c = 16.1 \text{ mm}$	$a = 7.4 \text{ mm}$ , $b = 14.3 \text{ mm}$ , $c = 16.1 \text{ mm}$
2	$a = 13.8 \text{ dm}$ , $c = 19.9 \text{ dm}$	$a = 13.8 \text{ dm}$ , $b = 14.4 \text{ dm}$ , $c = 19.9 \text{ dm}$
3	$b = 10.9 \text{ mm}$ , $c = 16 \text{ mm}$	$a = 11.7 \text{ mm}$ , $b = 10.9 \text{ mm}$ , $c = 16 \text{ mm}$
4	$a = 23.7 \text{ dm}$ , $c = 38.3 \text{ dm}$	$a = 23.7 \text{ dm}$ , $b = 30.1 \text{ dm}$ , $c = 38.3 \text{ dm}$
5	$b = 21.7 \text{ m}$ , $c = 22.5 \text{ m}$	$a = 5.8 \text{ m}$ , $b = 21.7 \text{ m}$ , $c = 22.5 \text{ m}$
6	$a = 22.4 \text{ mm}$ , $b = 12.7 \text{ mm}$	$a = 22.4 \text{ mm}$ , $b = 12.7 \text{ mm}$ , $c = 25.7 \text{ mm}$
7	$a = 23.7 \text{ cm}$ , $c = 46.3 \text{ cm}$	$a = 23.7 \text{ cm}$ , $b = 39.8 \text{ cm}$ , $c = 46.3 \text{ cm}$
8	$a = 36.6 \text{ m}$ , $b = 19.7 \text{ m}$	$a = 36.6 \text{ m}$ , $b = 19.7 \text{ m}$ , $c = 41.6 \text{ m}$
9	$a = 23.9 \text{ dm}$ , $b = 33.4 \text{ dm}$	$a = 23.9 \text{ dm}$ , $b = 33.4 \text{ dm}$ , $c = 41.1 \text{ dm}$
10	$a = 32.8 \text{ dm}$ , $b = 21.7 \text{ dm}$	$a = 32.8 \text{ dm}$ , $b = 21.7 \text{ dm}$ , $c = 39.3 \text{ dm}$
11	$b = 10.7 \text{ dm}$ , $c = 35.3 \text{ dm}$	$a = 33.6 \text{ dm}$ , $b = 10.7 \text{ dm}$ , $c = 35.3 \text{ dm}$
12	$a = 25.6 \text{ mm}$ , $c = 38.4 \text{ mm}$	$a = 25.6 \text{ mm}$ , $b = 28.6 \text{ mm}$ , $c = 38.4 \text{ mm}$

**Aufgabe 3:** Ein Dreieck hat die Seiten  $a$ ,  $b$ ,  $c$ . Untersuche auf Rechtwinkligkeit des Dreiecks.

Nr.	Gegeben:
1	$a = 12 \text{ cm}$ , $b = 5 \text{ cm}$ , $c = 13 \text{ cm}$
2	$a = 6 \text{ dm}$ , $b = 8 \text{ dm}$ , $c = 10 \text{ dm}$
3	$a = 2,4 \text{ mm}$ , $b = 3,2 \text{ mm}$ , $c = 4,1 \text{ mm}$
4	$a = 4 \text{ dm}$ , $b = 9 \text{ cm}$ , $c = 4,1 \text{ dm}$
5	$a = 1,6 \text{ m}$ , $b = 3 \text{ m}$ , $c = 3,4 \text{ m}$
6	$a = 12 \text{ cm}$ , $b = 35 \text{ cm}$ , $c = 3,8 \text{ dm}$
7	$a = 24 \text{ cm}$ , $b = 10 \text{ cm}$ , $c = 26 \text{ cm}$
8	$a = 15 \text{ m}$ , $b = 36 \text{ m}$ , $c = 40 \text{ m}$
9	$a = 0,5 \text{ m}$ , $b = 12 \text{ dm}$ , $c = 130 \text{ cm}$
10	$a = 20 \text{ cm}$ , $b = 21 \text{ cm}$ , $c = 30 \text{ cm}$

Lösungen:

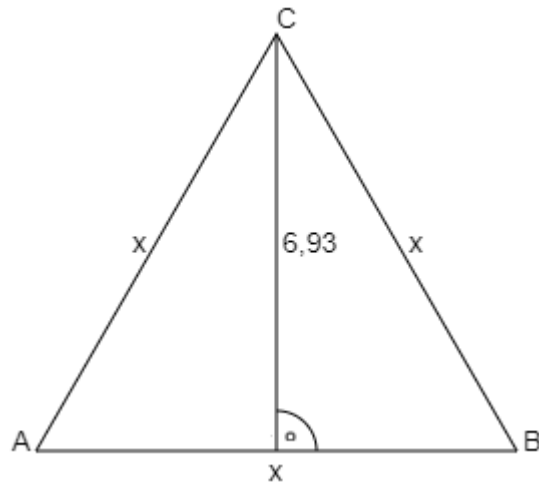
Nr.	Gegeben:	Lösungen:
1	$a = 12 \text{ cm}$ , $b = 5 \text{ cm}$ , $c = 13 \text{ cm}$	$12^2 + 5^2 = 13^2 \rightarrow$ Dreieck rechtwinklig
2	$a = 6 \text{ dm}$ , $b = 8 \text{ dm}$ , $c = 10 \text{ dm}$	$6^2 + 8^2 = 10^2 \rightarrow$ Dreieck rechtwinklig
3	$a = 2,4 \text{ mm}$ , $b = 3,2 \text{ mm}$ , $c = 4,1 \text{ mm}$	$2,4^2 + 3,2^2 \neq 4,1^2 \rightarrow$ kein rechter Winkel

4	a = 4 dm, b = 9 cm, c = 4,1 dm	$40^2+9^2 = 41^2$ (cm) -> Dreieck rechtwinklig
5	a = 1,6 m, b = 3 m, c = 3,4 m	$1,6^2+3^2 = 3,4^2$ -> Dreieck rechtwinklig
6	a = 12 cm, b = 35 cm, c = 3,8 dm	$12^2+35^2 \neq 38^2$ (cm) -> kein rechter Winkel
7	a = 24 cm, b = 10 cm, c = 26 cm	$24^2+10^2 = 26^2$ -> Dreieck rechtwinklig
8	a = 15 m, b = 36 m, c = 40 m	$15^2+36^2 \neq 40^2$ -> kein rechter Winkel
9	a = 0,5 m, b = 12 dm, c = 130 cm	$50^2+120^2 = 130^2$ (cm) -> Dreieck rechtwinklig
10	a = 20 cm, b = 21 cm, c = 31 cm	$20^2+21^2 \neq 31^2$ -> kein rechter Winkel

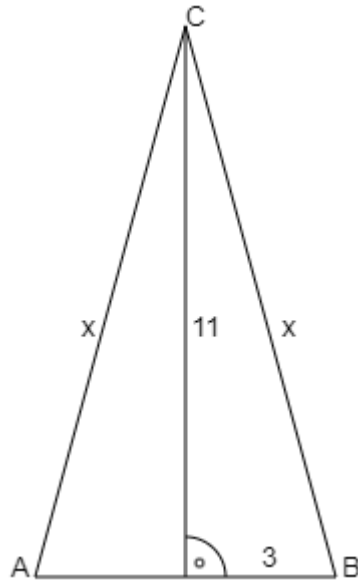
**Aufgabe 4:** Berechne die Seitenlängen x, y in den rechtwinkligen Dreiecken und den geometrischen Figuren (alle Zahlenangaben in cm).

Nr.	Grafik:
1	<p>A right-angled triangle with vertices A, B, and C. The right angle is at vertex C, indicated by a small square. Side BC is labeled 3, side AC is labeled 4, and the hypotenuse AB is labeled x.</p>
2	<p>A right-angled triangle with vertices A, B, and C. The right angle is at vertex C, indicated by a small square. Side AC is labeled x, side BC is labeled x, and the hypotenuse AB is labeled 8.5.</p>
3	<p>A right-angled triangle with vertices A, B, and C. The right angle is at vertex C, indicated by a small square. Side AC is labeled x+3, side BC is labeled x, and the hypotenuse AB is labeled 15.</p>

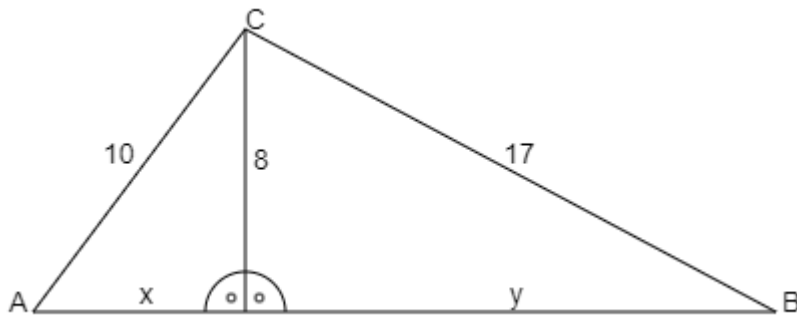
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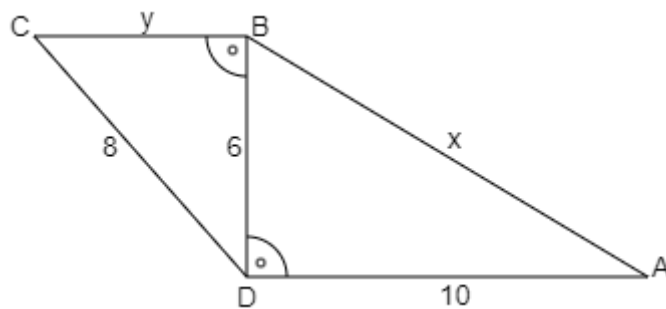
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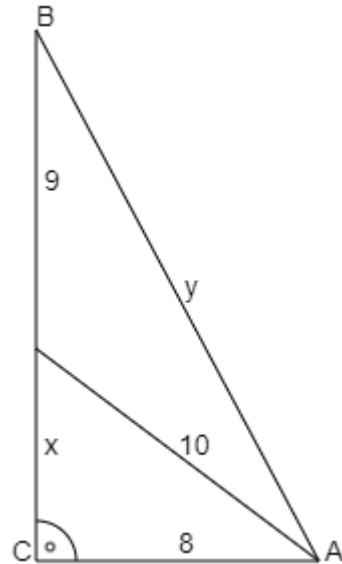
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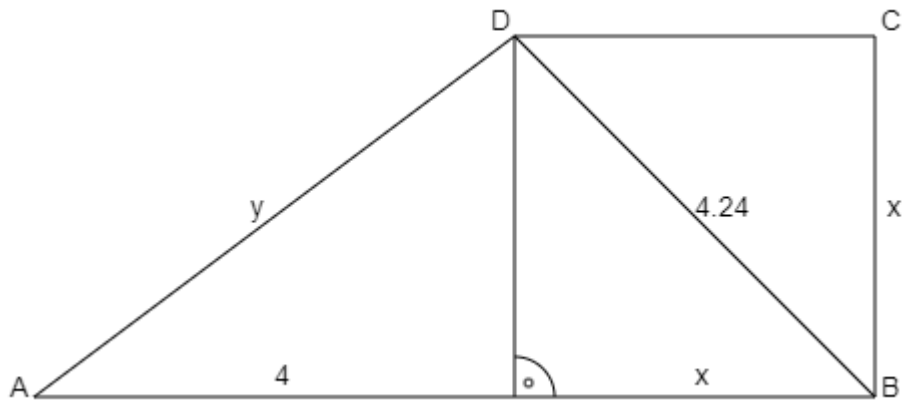
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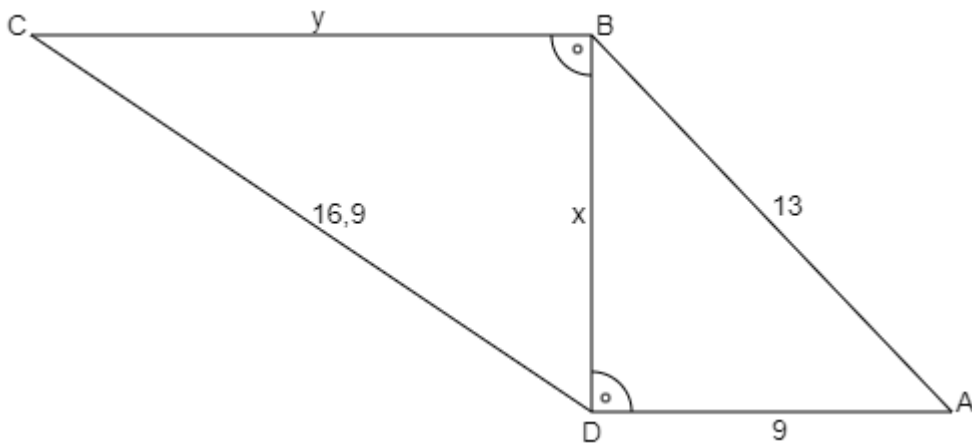
8



9



10



Lösungen: 1)  $x=5$  cm; 2)  $x=6$  cm; 3)  $x=13,9$  cm; 4)  $x=8$  cm; 5)  $x=11,4$  cm; 6)  $x=6$  cm,  $y=15$  cm; 7)  $x=11,7$  cm,  $y=5,3$  cm; 8)  $x=6$  cm,  $y=17$  cm; 9)  $x=3$  cm,  $y=5$  cm; 10)  $x=9,4$  cm,  $y=14$  cm.