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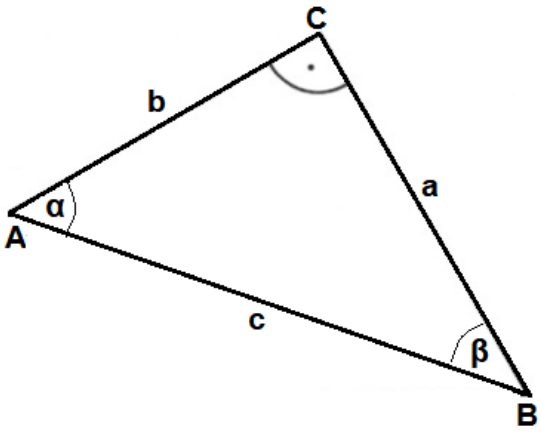
Mathematik-Aufgabenpool
> Trigonometrie II (rechtwinklige Dreiecke)

Einleitung: In einem rechtwinkligen Dreieck ΔABC mit den Seiten a, b, c und den Winkeln α, β, γ bei γ = 90° heißen a und b Katheten, c Hypotenuse. Die Kathete, die gegenüber einem Winkel α oder β liegt, heißt Gegenkathete (bei Winkel α Seite a, bei Winkel β Seite b), die Kathete, die an einem Winkel α oder β liegt, heißt Ankathete (bei Winkel α Seite b, bei Winkel β Seite a).

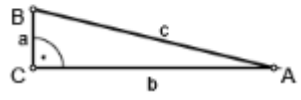
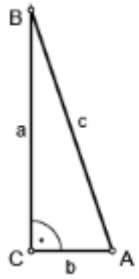
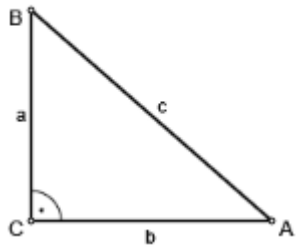
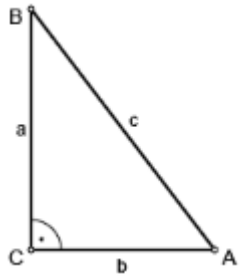
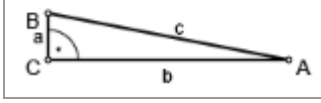


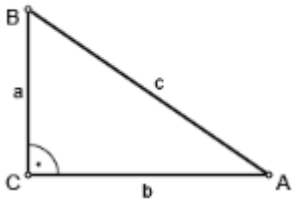
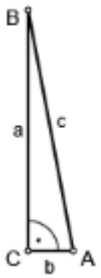
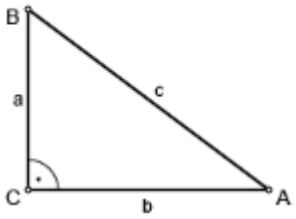
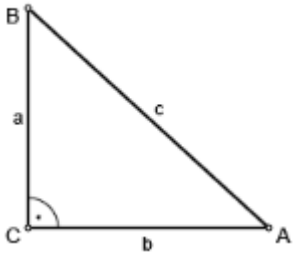
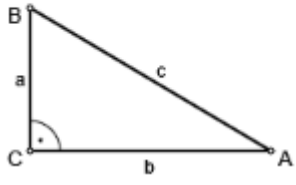
Rechtwinkliges Dreieck: Seiten a, b, c; Winkel α, β, γ=90°

Formelsammlung:

Satz des Pythagoras	$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2}$ (Hypotenuse) $a^2 = c^2 - b^2 \Rightarrow a = \sqrt{c^2 - b^2}$ (Kathete) $b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2}$ (Kathete)		
Trigonometrische Beziehungen (Sinus, Kosinus, Tangens)	$\sin\alpha = \frac{a}{c} = \frac{Gegenkathete}{Hypotenuse}$	$\cos\alpha = \frac{b}{c} = \frac{Ankathete}{Hypotenuse}$	$\tan\alpha = \frac{a}{b} = \frac{Gegenkathete}{Ankathete}$
	$\sin\beta = \frac{b}{c} = \frac{Gegenkathete}{Hypotenuse}$	$\cos\beta = \frac{a}{c} = \frac{Ankathete}{Hypotenuse}$	$\tan\beta = \frac{b}{a} = \frac{Gegenkathete}{Ankathete}$
Winkel	α + β = 90°	α = 90° – β	β = 90° – α
Umfang	u = a + b + c		
Fläche	$A = \frac{1}{2}ab$		

Aufgabe 1: Berechne die fehlenden Größen im rechtwinkligen Dreieck ΔABC (Winkel γ = 90°, a, b = Katheten, c = Hypotenuse, Winkel α, β, A = Flächeninhalt, u = Umfang).

Nr.	*Gegeben:*	*Grafik:*
1	a = 1.3 cm, β = 76.5°	
2	a = 9.8 cm, A = 16.2 cm^2	
3	b = 4.7 cm, c = 6.2 cm	
4	a = 5.5 cm, A = 11.6 cm^2	
5	a = 1.3 cm, A = 4.4 cm^2	

6	a = 5.7 cm, b = 8.3 cm	
7	a = 6.3 cm, β = 10.8°	
8	b = 3.2 cm, c = 4 cm	
9	a = 3.2 cm, β = 47.6°	
10	b = 2.7 cm, α = 30.7°	

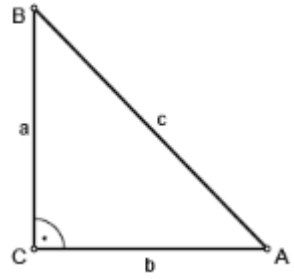
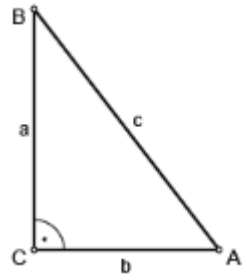
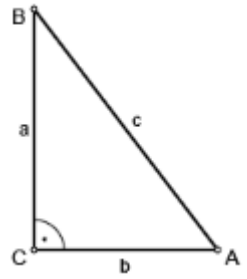
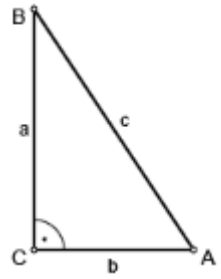
Vorgehensweise: Zur Ermittlung der fehlenden Größen beim rechtwinkligen Dreieck ist die obige Formelsammlung anzuwenden.

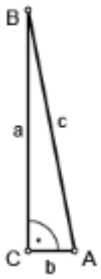
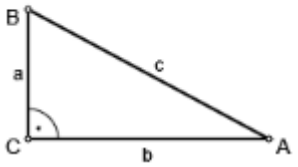
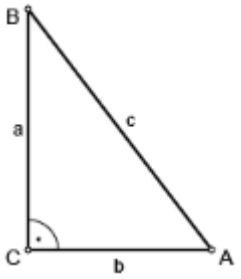
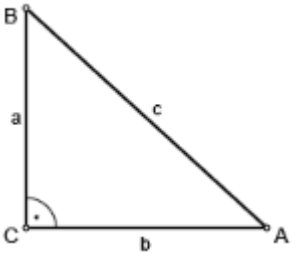
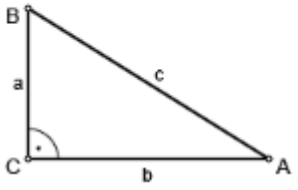
Lösungen:

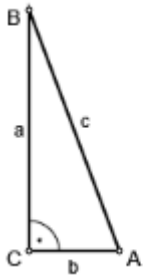
Nr.	*Gegeben:*	*Lösungen:*
1	a = 1.3 cm, β = 76.5°	a = 1.3 cm, b = 5.4 cm, c = 5.6 cm, α = 13.5°, β = 76.5°, A = 3.5 cm^2, u = 12.3 cm
2	a = 9.8 cm, A = 16.2 cm^2	a = 9.8 cm, b = 3.3 cm, c = 10.3 cm, α = 71.4°, β = 18.6°, A = 16.2 cm^2, u = 23.4 cm
3	b = 4.7 cm, c = 6.2 cm	a = 4.1 cm, b = 4.7 cm, c = 6.2 cm, α = 41.1°, β = 48.9°, A = 9.6 cm^2, u = 15 cm

4	a = 5.5 cm, A = 11.6 cm^2	a = 5.5 cm, b = 4.2 cm, c = 6.9 cm, α = 52.6°, β = 37.4°, A = 11.6 cm^2, u = 16.6 cm
5	a = 1.3 cm, A = 4.4 cm^2	a = 1.3 cm, b = 6.7 cm, c = 6.8 cm, α = 11°, β = 79°, A = 4.4 cm^2, u = 14.8 cm
6	a = 5.7 cm, b = 8.3 cm	a = 5.7 cm, b = 8.3 cm, c = 10.1 cm, α = 34.5°, β = 55.5°, A = 23.7 cm^2, u = 24.1 cm
7	a = 6.3 cm, β = 10.8°	a = 6.3 cm, b = 1.2 cm, c = 6.4 cm, α = 79.2°, β = 10.8°, A = 3.8 cm^2, u = 13.9 cm
8	b = 3.2 cm, c = 4 cm	a = 2.4 cm, b = 3.2 cm, c = 4 cm, α = 36.9°, β = 53.1°, A = 3.8 cm^2, u = 9.6 cm
9	a = 3.2 cm, β = 47.6°	a = 3.2 cm, b = 3.5 cm, c = 4.7 cm, α = 42.4°, β = 47.6°, A = 5.6 cm^2, u = 11.4 cm
10	b = 2.7 cm, α = 30.7°	a = 1.6 cm, b = 2.7 cm, c = 3.1 cm, α = 30.7°, β = 59.3°, A = 2.2 cm^2, u = 7.4 cm

Aufgabe 2: Berechne die fehlenden Größen im rechtwinkligen Dreieck ΔABC (Winkel γ = 90°, a, b = Katheten, c = Hypotenuse, Winkel α, β, A = Flächeninhalt, u = Umfang).

Nr.	*Gegeben:*	*Grafik:*
1	b = 6.6 dm, A = 22.4 dm^2	
2	b = 7.1 dm, β = 37.7°	
3	b = 6.7 m, c = 11.1 m	
4	c = 11.6 mm, α = 56.6°	

5	b = 1.7 dm, A = 7.4 dm^2	
6	b = 6.9 m, A = 12.8 m^2	
7	b = 5.4 dm, α = 52.7°	
8	b = 5.7 cm, A = 14.8 cm^2	
9	c = 1.9 cm, β = 58°	

10	a = 5.6 dm, c = 6 dm	

Vorgehensweise: Zur Ermittlung der fehlenden Größen beim rechtwinkligen Dreieck ist die obige Formelsammlung anzuwenden.

Lösungen:

Nr.	*Gegeben:*	*Lösungen:*
1	b = 6.6 dm, A = 22.4 dm^2	a = 6.8 dm, b = 6.6 dm, c = 9.5 dm, α = 45.9°, β = 44.1°, A = 22.4 dm^2, u = 22.9 dm
2	b = 7.1 dm, β = 37.7°	a = 9.2 dm, b = 7.1 dm, c = 11.6 dm, α = 52.3°, β = 37.7°, A = 32.7 dm^2, u = 27.9 dm
3	b = 6.7 m, c = 11.1 m	a = 8.8 m, b = 6.7 m, c = 11.1 m, α = 52.7°, β = 37.3°, A = 29.5 m^2, u = 26.6 m
4	c = 11.6 mm, α = 56.6°	a = 9.7 mm, b = 6.4 mm, c = 11.6 mm, α = 56.6°, β = 33.4°, A = 31 mm^2, u = 27.7 mm
5	b = 1.7 dm, A = 7.4 dm^2	a = 8.7 dm, b = 1.7 dm, c = 8.9 dm, α = 78.9°, β = 11.1°, A = 7.4 dm^2, u = 19.3 dm
6	b = 6.9 m, A = 12.8 m^2	a = 3.7 m, b = 6.9 m, c = 7.8 m, α = 28.2°, β = 61.8°, A = 12.8 m^2, u = 18.4 m
7	b = 5.4 dm, α = 52.7°	a = 7.1 dm, b = 5.4 dm, c = 8.9 dm, α = 52.7°, β = 37.3°, A = 19.2 dm^2, u = 21.4 dm
8	b = 5.7 cm, A = 14.8 cm^2	a = 5.2 cm, b = 5.7 cm, c = 7.7 cm, α = 42.4°, β = 47.6°, A = 14.8 cm^2, u = 18.6 cm
9	c = 1.9 cm, β = 58°	a = 1 cm, b = 1.6 cm, c = 1.9 cm, α = 32°, β = 58°, A = 0.8 cm^2, u = 4.5 cm
10	a = 5.6 dm, c = 6 dm	a = 5.6 dm, b = 2.1 dm, c = 6 dm, α = 69.4°, β = 20.6°, A = 5.9 dm^2, u = 13.7 dm

Aufgabe 3: Berechne die fehlenden Größen im rechtwinkligen Dreieck ΔABC (Winkel γ = 90°, a, b = Katheten, c = Hypotenuse, Winkel α, β, A = Flächeninhalt, u = Umfang).

Nr.	*Gegeben:*
1	b = 2.5 cm, c = 9.9 cm
2	b = 2 cm, A = 2 cm^2
3	a = 2.9 cm, A = 4.2 cm^2
4	b = 2.4 cm, β = 36.9°
5	a = 8.3 cm, β = 38.1°
6	a = 8.3 cm, b = 8.1 cm
7	a = 3.8 cm, α = 34.2°
8	a = 3.4 cm, α = 19.9°
9	a = 4.7 cm, b = 6.3 cm
10	b = 1.9 cm, A = 3.5 cm^2
11	a = 7.8 cm, β = 22.9°
12	b = 4.6 cm, c = 5.9 cm
13	a = 7.4 cm, c = 9.3 cm
14	c = 12.3 cm, β = 41.4°

15	b = 1 cm, α = 77.2°
16	a = 2.5 cm, c = 3.4 cm
17	b = 8.3 cm, c = 8.4 cm
18	a = 1.1 cm, α = 6.4°
19	b = 2.1 cm, c = 3.8 cm
20	c = 12.4 cm, β = 51.6°

Vorgehensweise: Zur Ermittlung der fehlenden Größen beim rechtwinkligen Dreieck ist die obige Formelsammlung anzuwenden.

Lösungen:

Nr.	*Gegeben:*	*Lösungen:*
1	b = 2.5 cm, c = 9.9 cm	a = 9.6 cm, b = 2.5 cm, c = 9.9 cm, α = 75.4°, β = 14.6°, A = 12 cm^2, u = 22 cm
2	b = 2 cm, A = 2 cm^2	a = 2 cm, b = 2 cm, c = 2.8 cm, α = 45°, β = 45°, A = 2 cm^2, u = 6.8 cm
3	a = 2.9 cm, A = 4.2 cm^2	a = 2.9 cm, b = 2.9 cm, c = 4.1 cm, α = 45°, β = 45°, A = 4.2 cm^2, u = 9.9 cm
4	b = 2.4 cm, β = 36.9°	a = 3.2 cm, b = 2.4 cm, c = 4 cm, α = 53.1°, β = 36.9°, A = 3.8 cm^2, u = 9.6 cm
5	a = 8.3 cm, β = 38.1°	a = 8.3 cm, b = 6.5 cm, c = 10.5 cm, α = 51.9°, β = 38.1°, A = 27 cm^2, u = 25.3 cm
6	a = 8.3 cm, b = 8.1 cm	a = 8.3 cm, b = 8.1 cm, c = 11.6 cm, α = 45.7°, β = 44.3°, A = 33.6 cm^2, u = 28 cm
7	a = 3.8 cm, α = 34.2°	a = 3.8 cm, b = 5.6 cm, c = 6.8 cm, α = 34.2°, β = 55.8°, A = 10.6 cm^2, u = 16.2 cm
8	a = 3.4 cm, α = 19.9°	a = 3.4 cm, b = 9.4 cm, c = 10 cm, α = 19.9°, β = 70.1°, A = 16 cm^2, u = 22.8 cm
9	a = 4.7 cm, b = 6.3 cm	a = 4.7 cm, b = 6.3 cm, c = 7.9 cm, α = 36.7°, β = 53.3°, A = 14.8 cm^2, u = 18.9 cm
10	b = 1.9 cm, A = 3.5 cm^2	a = 3.7 cm, b = 1.9 cm, c = 4.2 cm, α = 62.8°, β = 27.2°, A = 3.5 cm^2, u = 9.8 cm
11	a = 7.8 cm, β = 22.9°	a = 7.8 cm, b = 3.3 cm, c = 8.5 cm, α = 67.1°, β = 22.9°, A = 12.9 cm^2, u = 19.6 cm
12	b = 4.6 cm, c = 5.9 cm	a = 3.7 cm, b = 4.6 cm, c = 5.9 cm, α = 38.8°, β = 51.2°, A = 8.5 cm^2, u = 14.2 cm
13	a = 7.4 cm, c = 9.3 cm	a = 7.4 cm, b = 5.7 cm, c = 9.3 cm, α = 52.4°, β = 37.6°, A = 21.1 cm^2, u = 22.4 cm
14	c = 12.3 cm, β = 41.4°	a = 9.2 cm, b = 8.1 cm, c = 12.3 cm, α = 48.6°, β = 41.4°, A = 37.3 cm^2, u = 29.6 cm
15	b = 1 cm, α = 77.2°	a = 4.4 cm, b = 1 cm, c = 4.5 cm, α = 77.2°, β = 12.8°, A = 2.2 cm^2, u = 9.9 cm
16	a = 2.5 cm, c = 3.4 cm	a = 2.5 cm, b = 2.3 cm, c = 3.4 cm, α = 47.4°, β = 42.6°, A = 2.9 cm^2, u = 8.2 cm
17	b = 8.3 cm, c = 8.4 cm	a = 1.5 cm, b = 8.3 cm, c = 8.4 cm, α = 10.2°, β = 79.8°, A = 6.2 cm^2, u = 18.2 cm
18	a = 1.1 cm, α = 6.4°	a = 1.1 cm, b = 9.8 cm, c = 9.9 cm, α = 6.4°, β = 83.6°, A = 5.4 cm^2, u = 20.8 cm
19	b = 2.1 cm, c = 3.8 cm	a = 3.2 cm, b = 2.1 cm, c = 3.8 cm, α = 56.7°, β = 33.3°, A = 3.4 cm^2, u = 9.1 cm
20	c = 12.4 cm, β = 51.6°	a = 7.7 cm, b = 9.7 cm, c = 12.4 cm, α = 38.4°, β = 51.6°, A = 37.3 cm^2, u = 29.8 cm

Aufgabe 4: Berechne die fehlenden Größen im rechtwinkligen Dreieck ΔABC (Winkel γ = 90°, a, b = Katheten, c = Hypotenuse, Winkel α, β, A = Flächeninhalt, u = Umfang).

Nr.	*Gegeben:*
1	a = 19.4 dm, c = 25.3 dm
2	a = 14.2 m, α = 40.9°
3	a = 15.1 cm, α = 58.6°
4	b = 17.7 dm, α = 41.4°
5	c = 15.9 m, β = 62.6°
6	a = 10.5 cm, A = 77.7 cm^2
7	b = 15.5 m, c = 19.3 m

8	a = 11.5 m, A = 73 m^2
9	a = 7.9 cm, α = 43.2°
10	c = 13.1 cm, α = 31.9°
11	a = 18 dm, A = 137.7 dm^2
12	b = 8.2 mm, α = 57.4°
13	a = 18.9 cm, A = 135.1 cm^2
14	b = 5.8 dm, β = 24°
15	a = 8.9 cm, A = 72.1 cm^2
16	b = 7.7 cm, c = 12.5 cm
17	c = 17 cm, α = 57.2°
18	b = 13.9 cm, α = 27.7°
19	a = 17.6 dm, β = 33.2°
20	a = 17.9 m, β = 22.5°

Vorgehensweise: Zur Ermittlung der fehlenden Größen beim rechtwinkligen Dreieck ist die obige Formelsammlung anzuwenden.

Lösungen:

Nr.	*Gegeben:*	*Lösungen:*
1	a = 19.4 dm, c = 25.3 dm	a = 19.4 dm, b = 16.3 dm, c = 25.3 dm, α = 50°, β = 40°, A = 158.1 dm^2, u = 61 dm
2	a = 14.2 m, α = 40.9°	a = 14.2 m, b = 16.4 m, c = 21.7 m, α = 40.9°, β = 49.1°, A = 116.4 m^2, u = 52.3 m
3	a = 15.1 cm, α = 58.6°	a = 15.1 cm, b = 9.2 cm, c = 17.7 cm, α = 58.6°, β = 31.4°, A = 69.5 cm^2, u = 42 cm
4	b = 17.7 dm, α = 41.4°	a = 15.6 dm, b = 17.7 dm, c = 23.6 dm, α = 41.4°, β = 48.6°, A = 138.1 dm^2, u = 56.9 dm
5	c = 15.9 m, β = 62.6°	a = 7.3 m, b = 14.1 m, c = 15.9 m, α = 27.4°, β = 62.6°, A = 51.5 m^2, u = 37.3 m
6	a = 10.5 cm, A = 77.7 cm^2	a = 10.5 cm, b = 14.8 cm, c = 18.1 cm, α = 35.4°, β = 54.6°, A = 77.7 cm^2, u = 43.4 cm
7	b = 15.5 m, c = 19.3 m	a = 11.5 m, b = 15.5 m, c = 19.3 m, α = 36.6°, β = 53.4°, A = 89.1 m^2, u = 46.3 m
8	a = 11.5 m, A = 73 m^2	a = 11.5 m, b = 12.7 m, c = 17.1 m, α = 42.2°, β = 47.8°, A = 73 m^2, u = 41.3 m
9	a = 7.9 cm, α = 43.2°	a = 7.9 cm, b = 8.4 cm, c = 11.5 cm, α = 43.2°, β = 46.8°, A = 33.2 cm^2, u = 27.8 cm
10	c = 13.1 cm, α = 31.9°	a = 6.9 cm, b = 11.1 cm, c = 13.1 cm, α = 31.9°, β = 58.1°, A = 38.3 cm^2, u = 31.1 cm
11	a = 18 dm, A = 137.7 dm^2	a = 18 dm, b = 15.3 dm, c = 23.6 dm, α = 49.6°, β = 40.4°, A = 137.7 dm^2, u = 56.9 dm
12	b = 8.2 mm, α = 57.4°	a = 12.8 mm, b = 8.2 mm, c = 15.2 mm, α = 57.4°, β = 32.6°, A = 52.5 mm^2, u = 36.2 mm
13	a = 18.9 cm, A = 135.1 cm^2	a = 18.9 cm, b = 14.3 cm, c = 23.7 cm, α = 52.9°, β = 37.1°, A = 135.1 cm^2, u = 56.9 cm
14	b = 5.8 dm, β = 24°	a = 13 dm, b = 5.8 dm, c = 14.2 dm, α = 66°, β = 24°, A = 37.7 dm^2, u = 33 dm
15	a = 8.9 cm, A = 72.1 cm^2	a = 8.9 cm, b = 16.2 cm, c = 18.5 cm, α = 28.8°, β = 61.2°, A = 72.1 cm^2, u = 43.6 cm
16	b = 7.7 cm, c = 12.5 cm	a = 9.8 cm, b = 7.7 cm, c = 12.5 cm, α = 51.8°, β = 38.2°, A = 37.7 cm^2, u = 30 cm
17	c = 17 cm, α = 57.2°	a = 14.3 cm, b = 9.2 cm, c = 17 cm, α = 57.2°, β = 32.8°, A = 65.8 cm^2, u = 40.5 cm
18	b = 13.9 cm, α = 27.7°	a = 7.3 cm, b = 13.9 cm, c = 15.7 cm, α = 27.7°, β = 62.3°, A = 50.7 cm^2, u = 36.9 cm
19	a = 17.6 dm, β = 33.2°	a = 17.6 dm, b = 11.5 dm, c = 21 dm, α = 56.8°, β = 33.2°, A = 101.2 dm^2, u = 50.1 dm
20	a = 17.9 m, β = 22.5°	a = 17.9 m, b = 7.4 m, c = 19.4 m, α = 67.5°, β = 22.5°, A = 66.2 m^2, u = 44.7 m