

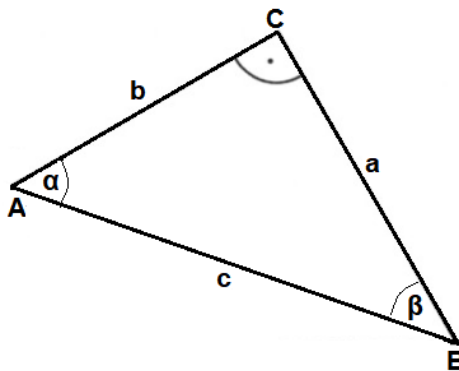
# Mathematik-Aufgabenpool

## > Satz des Pythagoras II

**Einleitung:** In einem rechtwinkligen Dreieck  $\triangle ABC$  mit den Seiten  $a, b, c$  und den Winkeln  $\alpha, \beta, \gamma$  bei  $\gamma = 90^\circ$  heißen  $a$  und  $b$  Katheten,  $c$  Hypotenuse. Beim gleichschenkligen Dreieck sind die zwei Schenkel  $a, b$  des Dreiecks gleich lang mit:  $a = b$ ; es kann durch die Höhe  $h$  zur Basisseite  $c$  in zwei rechtwinklige Dreiecke halbiert werden. Gleichseitige Dreiecke sind Dreiecke mit drei gleich langen Seiten  $a$ ; gleichseitige Dreiecke sind gleichschenklig. Darüber hinaus gelten die geometrischen Beziehungen in Vierecken  $ABCD$  (Trapeze, Parallelogramme, Rauten, Drachen, Rechtecke, Quadrate).

**Formelsammlung:**

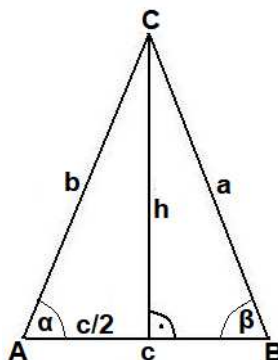
**a) Rechtwinklige Dreiecke:**



**Rechtwinkliges Dreieck:** Seiten  $a, b, c$ ; Winkel  $\alpha, \beta, \gamma=90^\circ$

|                     |   |
|---------------------|---|
| Satz des Pythagoras | $c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2} \text{ (Hypotenuse)}$ $a^2 = c^2 - b^2 \Rightarrow a = \sqrt{c^2 - b^2} \text{ (Kathete)}$ $b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2} \text{ (Kathete)}$ |
| Umfang              | $u = a + b + c$   |
| Flächeninhalt       | $A = \frac{1}{2} ab$  |

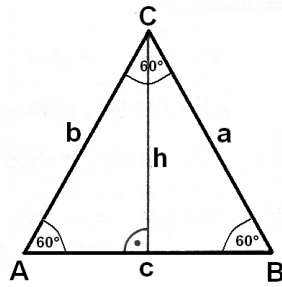
**b) Gleichschenklige Dreiecke:**



**Gleichschenkliges Dreieck:** Seiten  $a=b, c$ ; Basiswinkel  $\alpha=\beta$ , Winkel  $\gamma$ , Höhe  $h$

|                     |  |
|---------------------|--|
| Satz des Pythagoras | $h = \sqrt{a^2 - \left(\frac{c}{2}\right)^2} \text{ (Höhe)}$ |
| Umfang              | $u = 2a + c$   |
| Flächeninhalt       | $A = \frac{1}{2} ch$   |

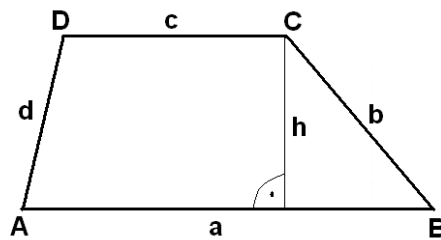
c) Gleichseitige Dreiecke:



**Gleichseitiges Dreieck:** Seiten  $a=b=c$ ; Winkel  $\alpha=\beta=\gamma$ , Höhe  $h$

|                     |                                   |
|---------------------|-----------------------------------|
| Satz des Pythagoras | $h = \frac{a}{2} \sqrt{3}$ (Höhe) |
| Umfang              | $u = 3a$                          |
| Flächeninhalt       | $A = \frac{a^2 \sqrt{3}}{4}$      |

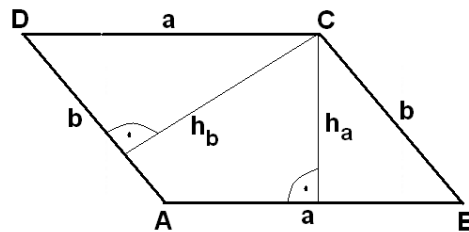
d) Trapeze:



**Trapez:** Parallele Seiten  $a, c$ , Höhe  $h$

|               |                               |
|---------------|-------------------------------|
| Umfang        | $u = a + b + c + d$           |
| Flächeninhalt | $A = \frac{a + c}{2} \cdot h$ |

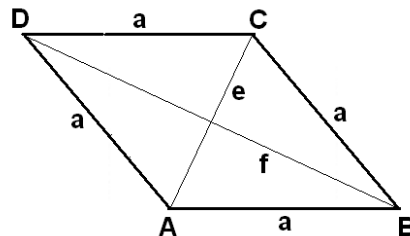
e) Parallelogramme:



**Parallelogramm:** Parallele Seiten  $a$  bzw.  $b$ , Höhen  $h_a, h_b$

|               |                                 |
|---------------|---------------------------------|
| Umfang        | $u = 2a + 2b$                   |
| Flächeninhalt | $A = a \cdot h_a = b \cdot h_b$ |

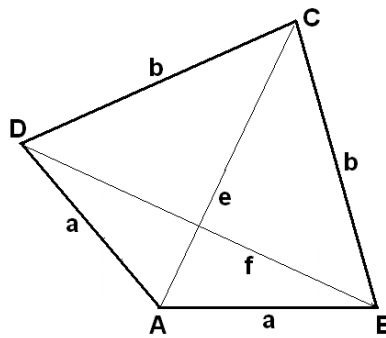
f) Rauten:



**Raute:** Seite  $a$ , Diagonalen  $e, f$

|                     |   |
|---------------------|---|
| Satz des Pythagoras | $\left(\frac{e}{2}\right)^2 + \left(\frac{f}{2}\right)^2 = a^2$ (Seite, halbe Diagonalen) |
| Umfang              | $u = 4a$  |
| Flächeninhalt       | $A = \frac{ef}{2}$  |

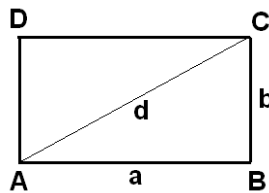
**g) Drachen:**



**Drache:** Gleich lange Seiten a bzw. b, Diagonale e, f

|               |                    |
|---------------|--------------------|
| Umfang        | $u = 2a + 2b$      |
| Flächeninhalt | $A = \frac{ef}{2}$ |

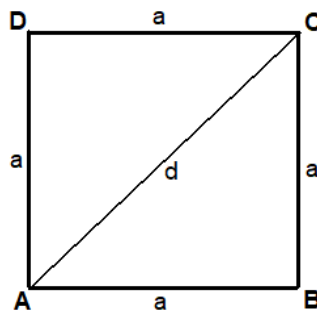
**h) Rechtecke:**



**Rechteck:** Parallele Seiten a bzw. b, Diagonale d

|                     |                                    |
|---------------------|------------------------------------|
| Satz des Pythagoras | $d = \sqrt{a^2 + b^2}$ (Diagonale) |
| Umfang              | $u = 2a + 2b$                      |
| Flächeninhalt       | $A = ab$                           |

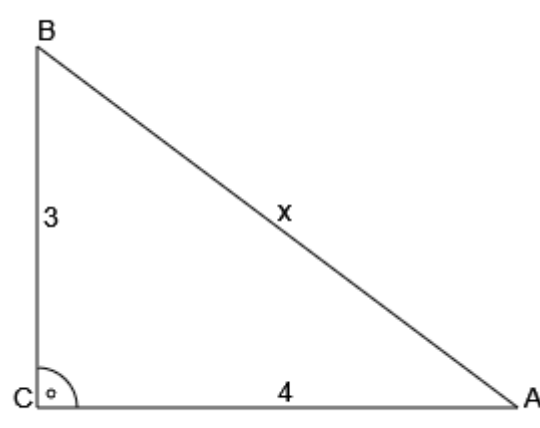
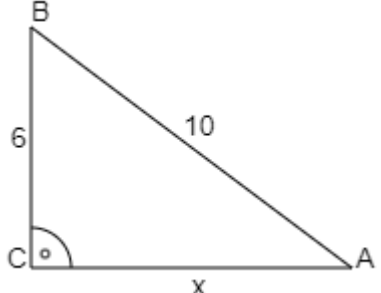
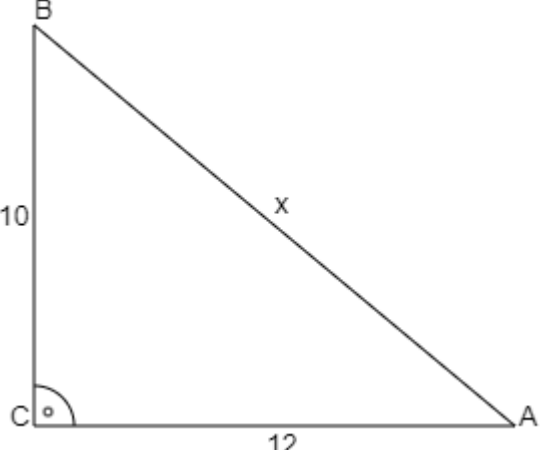
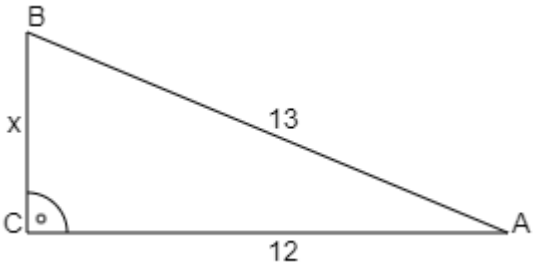
**i) Quadrate:**



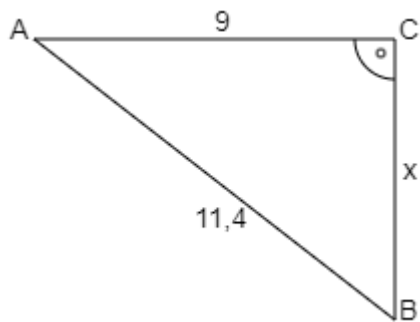
**Quadrat:** Seite a, Diagonale d

|                     |                             |
|---------------------|-----------------------------|
| Satz des Pythagoras | $d = a\sqrt{2}$ (Diagonale) |
| Umfang              | $u = 4a$                    |
| Flächeninhalt       | $A = a^2$                   |

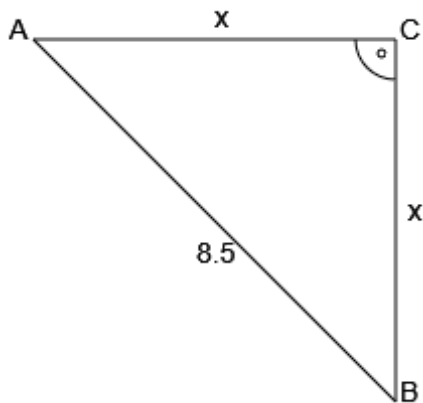
**Aufgabe 1:** Berechne die Seitenlänge  $x$  in den rechtwinkligen Dreiecken sowie Umfang und Flächeninhalt (alle Zahlenangaben in cm).

| Nr. | Grafik:   |
|-----|---|
| 1   |  <p>A right-angled triangle with vertices B at the top, C at the bottom-left, and A at the bottom-right. The right angle is at vertex C, indicated by a small square. The vertical leg BC is labeled with the number 3. The horizontal leg CA is labeled with the number 4. The hypotenuse AB is labeled with the variable <math>x</math>.</p>     |
| 2   |  <p>A right-angled triangle with vertices B at the top, C at the bottom-left, and A at the bottom-right. The right angle is at vertex C, indicated by a small square. The vertical leg BC is labeled with the number 6. The hypotenuse AB is labeled with the number 10. The horizontal leg CA is labeled with the variable <math>x</math>.</p>   |
| 3   |  <p>A right-angled triangle with vertices B at the top, C at the bottom-left, and A at the bottom-right. The right angle is at vertex C, indicated by a small square. The vertical leg BC is labeled with the number 10. The horizontal leg CA is labeled with the number 12. The hypotenuse AB is labeled with the variable <math>x</math>.</p> |
| 4   |  <p>A right-angled triangle with vertices B at the top, C at the bottom-left, and A at the bottom-right. The right angle is at vertex C, indicated by a small square. The vertical leg BC is labeled with the variable <math>x</math>. The horizontal leg CA is labeled with the number 12. The hypotenuse AB is labeled with the number 13.</p> |

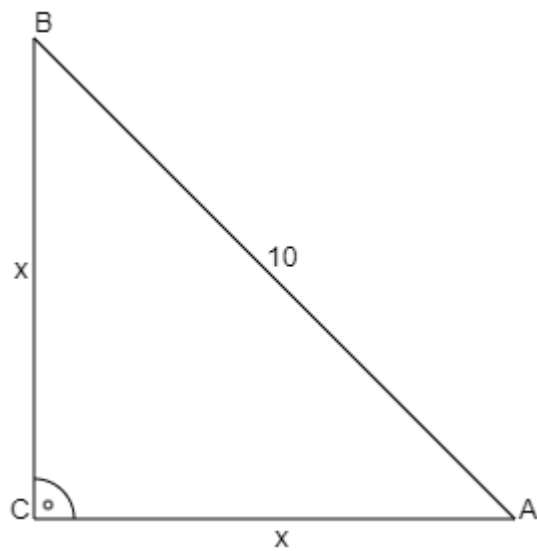
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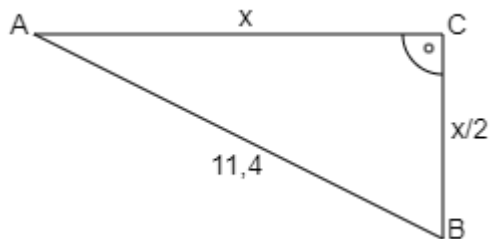
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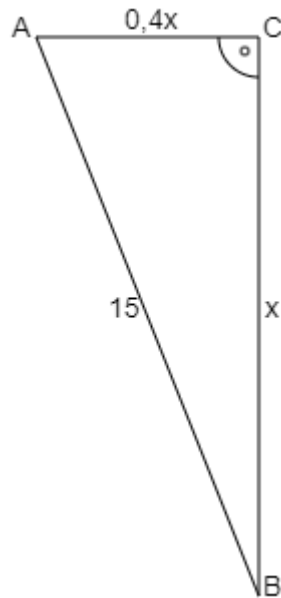
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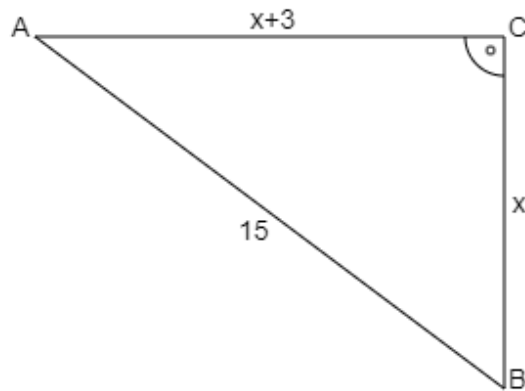
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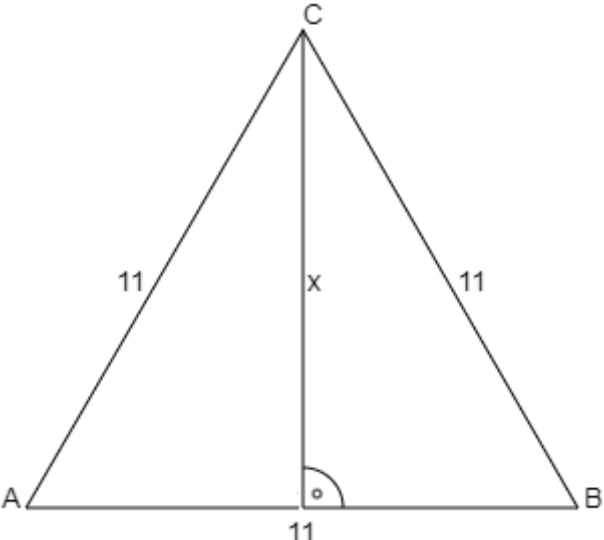
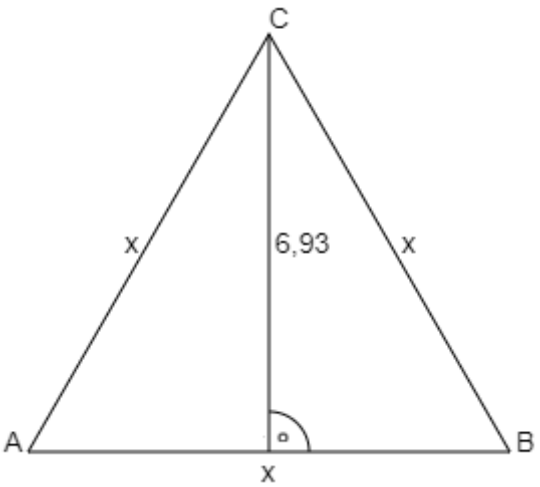
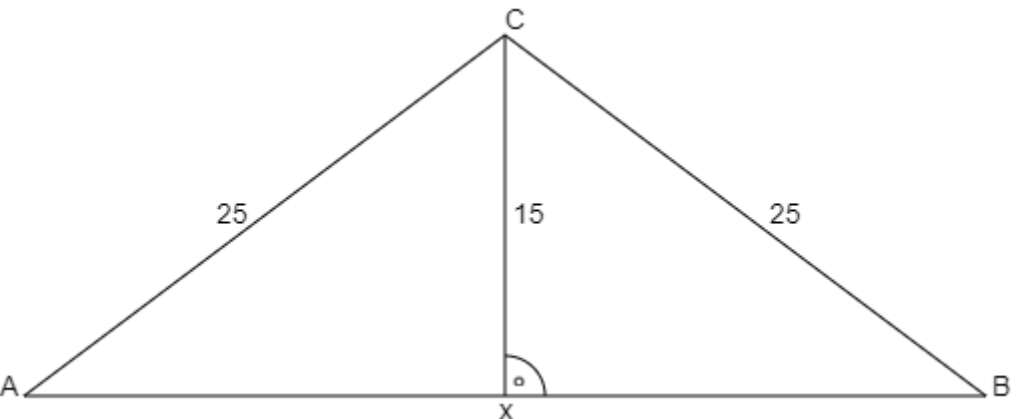
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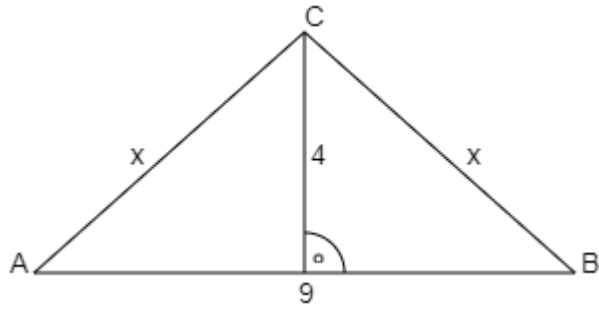
**Vorgehensweise:** Zur Ermittlung der fehlenden Größen in rechtwinkligen Dreiecken ist die obige Formelsammlung zu den rechtwinkligen Dreiecken anzuwenden.

**Lösungen:** 1)  $x=5$  cm,  $u = 12$  cm,  $A = 6$  cm<sup>2</sup>; 2)  $x = 8$  cm,  $u = 24$  cm,  $A = 24$  cm<sup>2</sup>; 3)  $x = 15,6$  cm,  $u = 37,6$  cm,  $A = 60$  cm<sup>2</sup>; 4)  $x = 5$  cm,  $u = 30$  cm,  $A = 30$  cm<sup>2</sup>; 5)  $x = 7$  cm,  $u = 27,4$  cm,  $A = 31,5$  cm<sup>2</sup>; 6)  $x = 6$  cm,  $u = 20,5$  cm,  $A = 18$  cm<sup>2</sup>; 7)  $x = 7,1$  cm,  $u = 24,2$  cm,  $A = 25$  cm<sup>2</sup>; 8)  $x = 10,2$  cm,  $u = 16,7$ ,  $A = 26$  cm<sup>2</sup>; 9)  $x = 13,9$  cm,  $u = 34,5$  cm,  $A = 38,6$  cm<sup>2</sup>; 10)  $x = 9$  cm,  $u = 36$  cm,  $A = 54$  cm<sup>2</sup>.

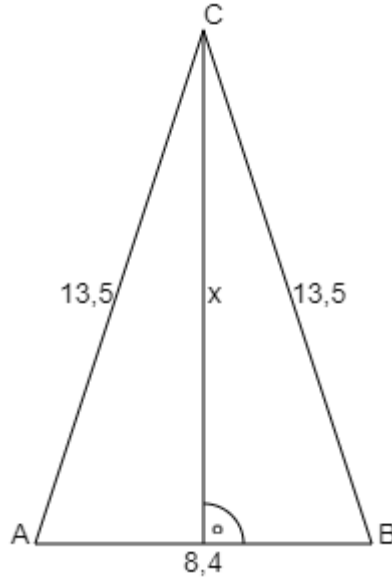
**Aufgabe 2:** Berechne die Länge  $x$  in den gleichseitigen und gleichschenkligen Dreiecken sowie deren Umfang und Flächeninhalt (alle Zahlenangaben in cm).

| Nr. | Grafik:  |
|-----|--|
| 1   |   |
| 2   |  |
| 3   |  |

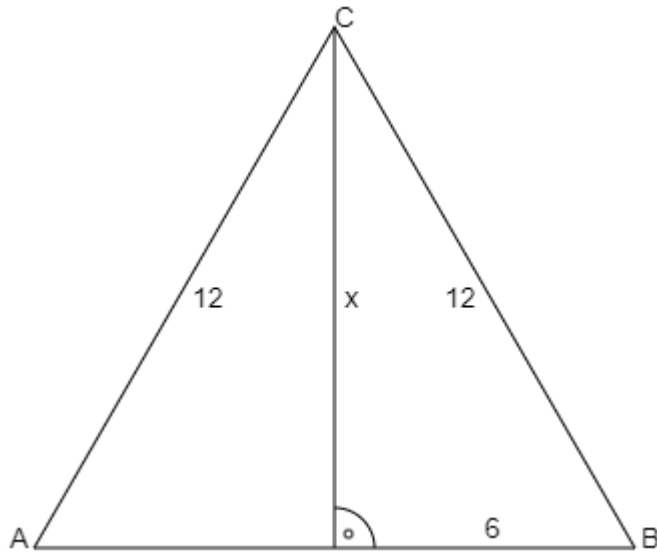
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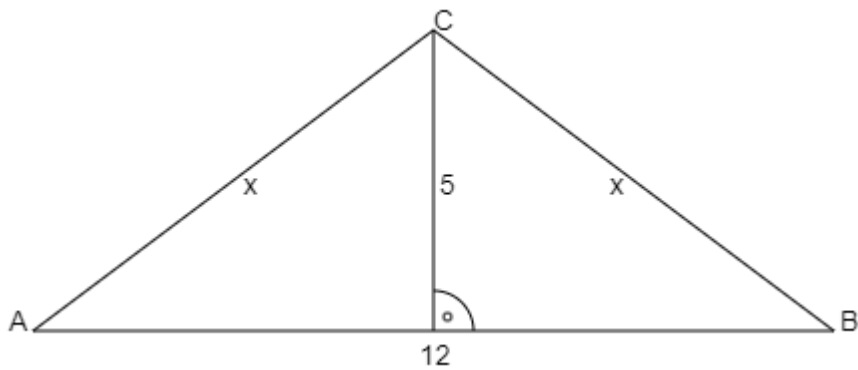
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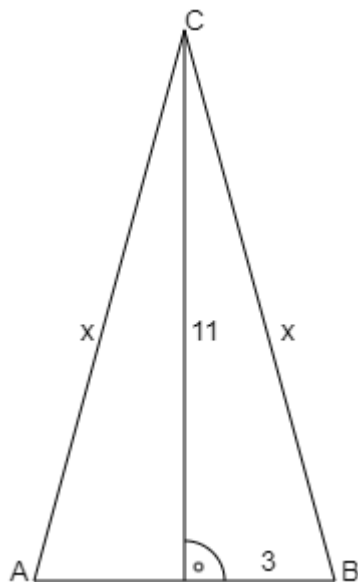


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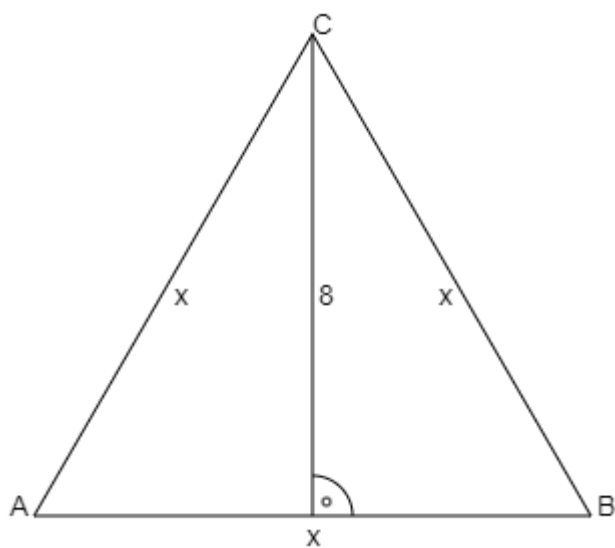




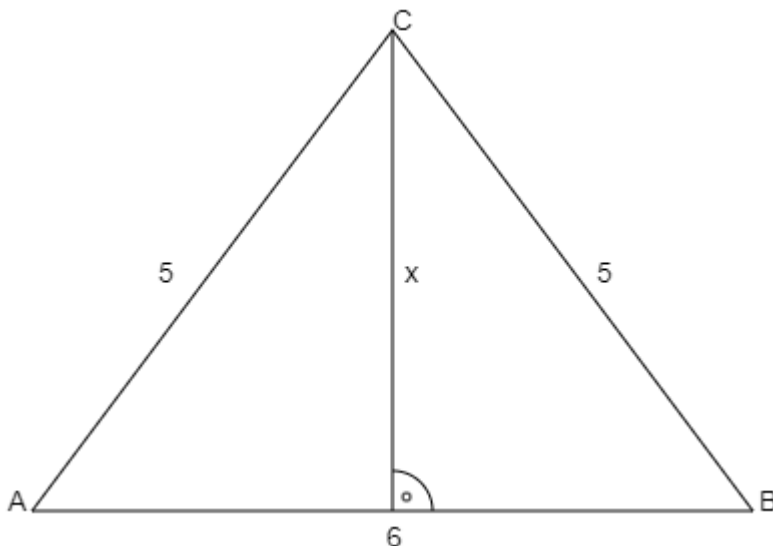
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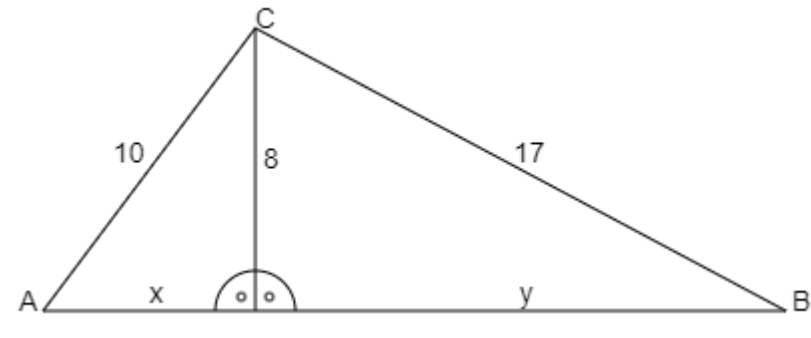
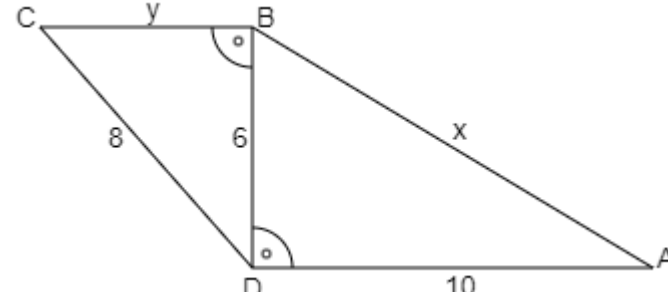
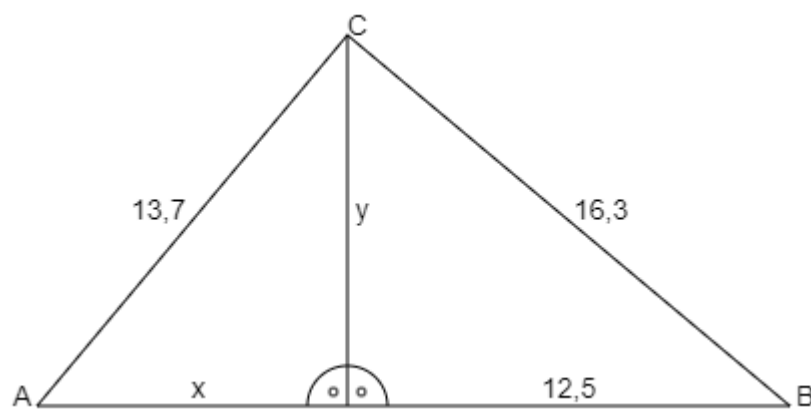
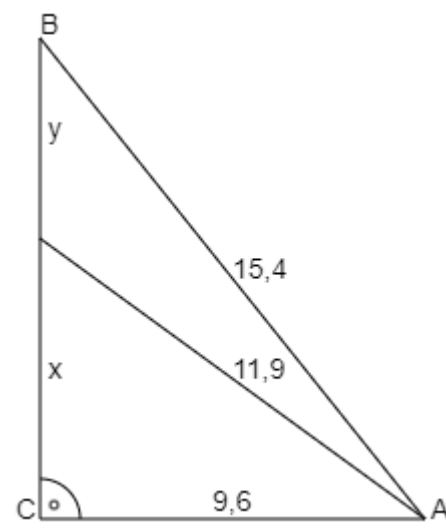
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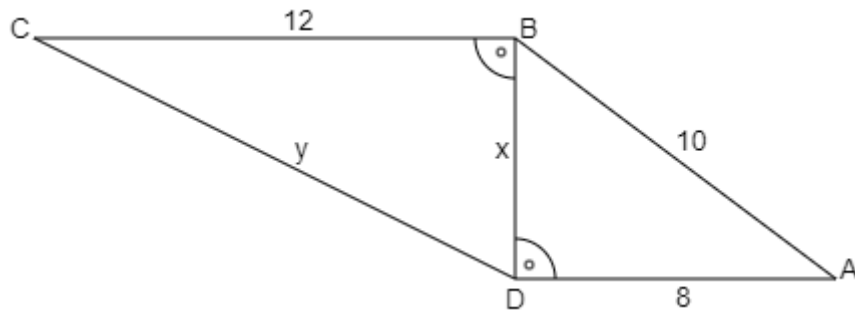
**Vorgehensweise:** Zur Ermittlung der fehlenden Größen in rechtwinkligen Dreiecken ist die obige Formelsammlung zu rechtwinkligen, gleichschenkligen und gleichseitigen Dreiecken anzuwenden. Zusätzlich

**Lösungen:** 1)  $x = 9,53$  cm,  $u = 33$  cm,  $A = 38,12$  cm<sup>2</sup>; 2)  $x = 8$  cm,  $u = 24$  cm,  $A = 27,71$  cm<sup>2</sup>; 3)  $x = 40$  cm,  $u = 90$  cm,  $A = 300$  cm<sup>2</sup>; 4)  $x = 6$  cm,  $u = 21$  cm,  $A = 18$  cm<sup>2</sup>; 5)  $x = 12,8$  cm,  $u = 35,4$  cm,  $A = 53,8$  cm<sup>2</sup>; 6)  $x = 10,4$  cm,  $u = 36$  cm,  $A = 62,4$  cm<sup>2</sup>; 7)  $x = 7,8$  cm,  $u = 27,6$  cm,  $A = 30$  cm<sup>2</sup>; 8)  $x = 11,4$  cm,  $u = 28,8$  cm,  $A = 33$  cm<sup>2</sup>; 9)  $x = 9,24$  cm,  $u = 27,72$  cm,  $A = 36,96$  cm<sup>2</sup>; 10)  $x = 4$  cm,  $u = 16$  cm,  $A = 12$  cm<sup>2</sup>.

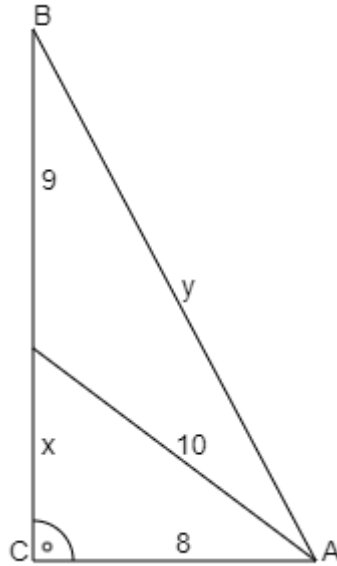
**Aufgabe 3:** Berechne die Seiten-/Streckenlängen  $x$  und  $y$  in den geometrischen Figuren (alle Zahlenangaben in cm).

| Nr. | Grafik:   |
|-----|---|
| 1   |  <p>A right-angled triangle <math>ABC</math> with the right angle at <math>D</math> on the base <math>AB</math>. The altitude <math>CD</math> has a length of 8. The side <math>AC</math> has a length of 10, and the side <math>CB</math> has a length of 17. The segments of the base are <math>AD = x</math> and <math>DB = y</math>.</p>  |
| 2   |  <p>A right-angled triangle <math>ABC</math> with the right angle at <math>D</math> on the side <math>BC</math>. The altitude <math>BD</math> has a length of 6. The side <math>BC</math> has a length of 8, and the side <math>DA</math> has a length of 10. The side <math>AB</math> has a length of <math>x</math>, and the segment <math>CD</math> has a length of <math>y</math>.</p>             |
| 3   |  <p>A right-angled triangle <math>ABC</math> with the right angle at <math>D</math> on the base <math>AB</math>. The altitude <math>CD</math> has a length of <math>y</math>. The side <math>AC</math> has a length of 13,7, and the side <math>CB</math> has a length of 16,3. The segment <math>DB</math> has a length of 12,5, and the segment <math>AD</math> has a length of <math>x</math>.</p> |
| 4   |  <p>A right-angled triangle <math>ABC</math> with the right angle at <math>C</math>. The altitude <math>CD</math> has a length of <math>x</math>. The side <math>CB</math> has a length of <math>y</math>, and the side <math>AB</math> has a length of 15,4. The side <math>AC</math> has a length of 9,6, and the segment <math>AD</math> has a length of 11,9.</p>                                  |

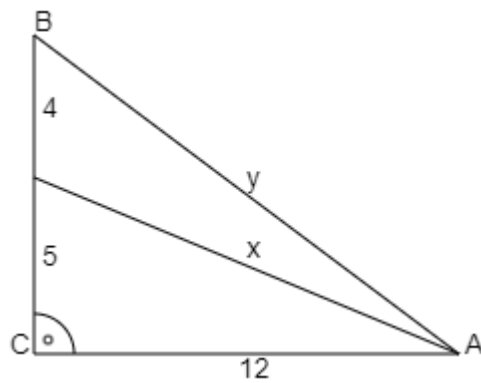
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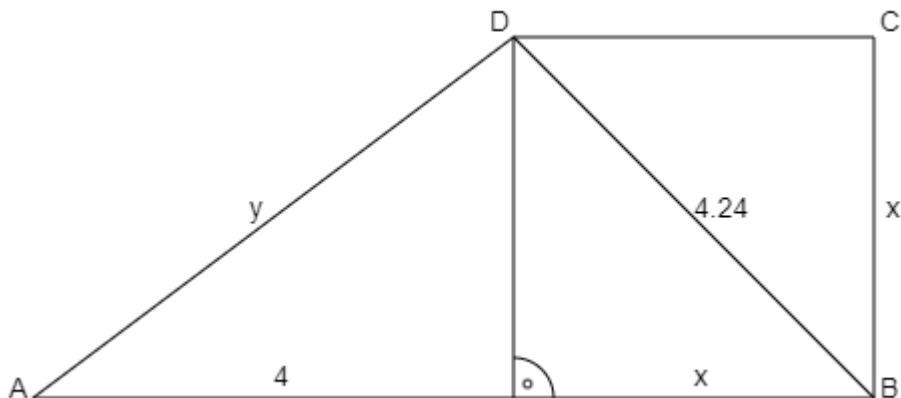
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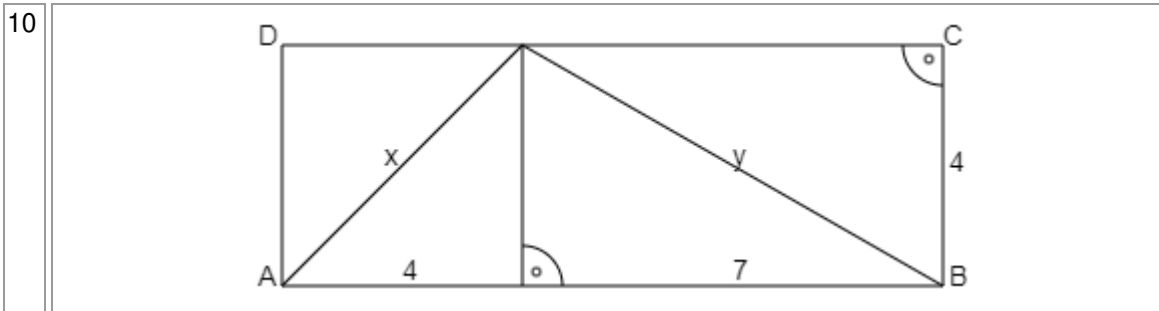
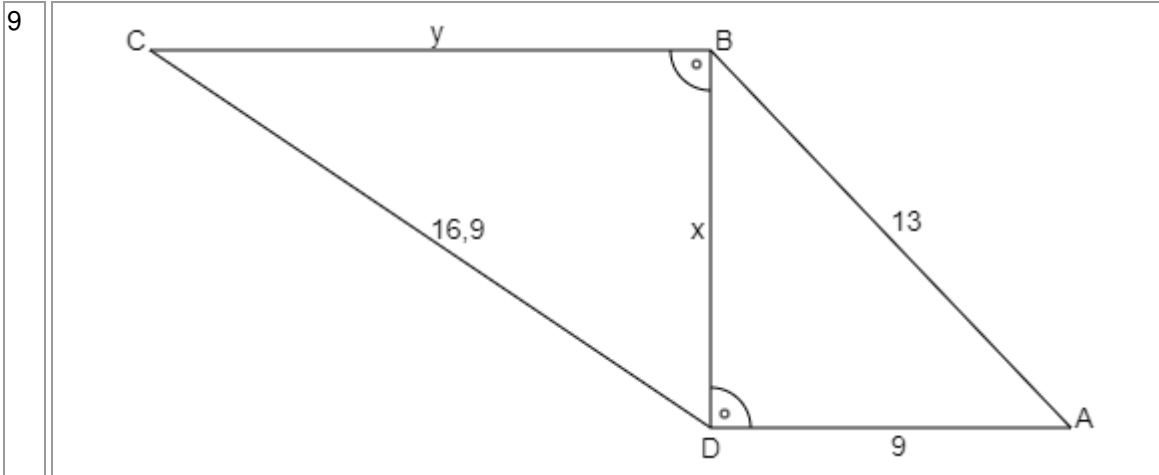


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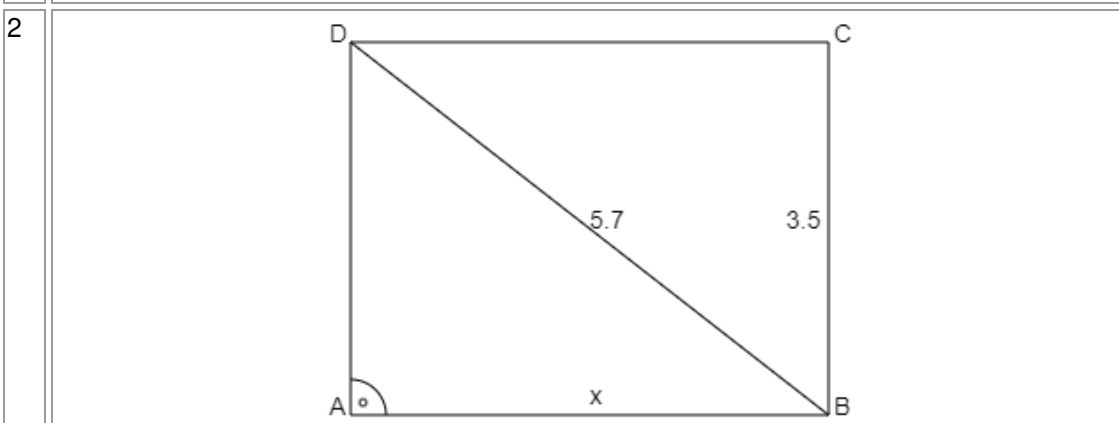
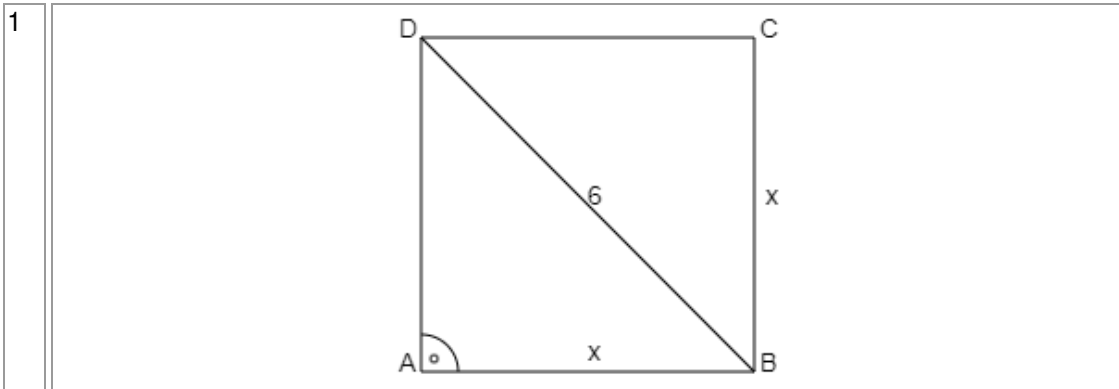


**Vorgehensweise:** Zur Ermittlung der fehlenden Größen bei den Vierecken ist die obige Formelsammlung anzuwenden.

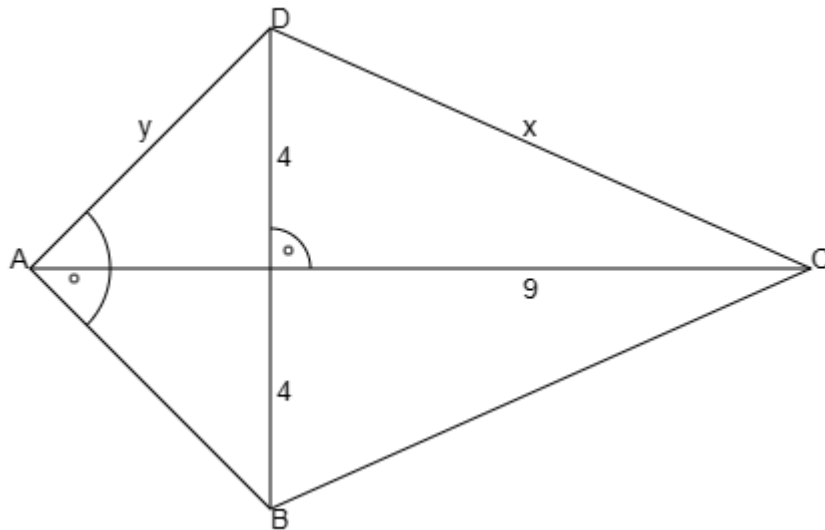
**Lösungen:** 1)  $x = 6$  cm,  $y = 15$  cm; 2)  $x = 11,7$  cm,  $y = 5,3$  cm; 3)  $x = 8,8$  cm,  $y = 10,5$  cm; 4)  $x = 7$  cm,  $y = 5$  cm;  
 5)  $x = 6$  cm,  $y = 13,4$  cm; 6)  $x = 6$  cm,  $y = 17$  cm; 7)  $x = 13$  cm,  $y = 15$  cm; 8)  $x = 3$  cm,  $y = 5$  cm; 9)  $x = 9,4$  cm,  $y = 14$  cm;  
 10)  $x = 5,66$  cm,  $y = 8,06$  cm.

**Aufgabe 4:** Berechne die Seiten-/Streckenlängen  $x$ ,  $y$  und  $z$  sowie Umfang und Flächeninhalt der folgenden Vierecke (alle Zahlenangaben in cm).

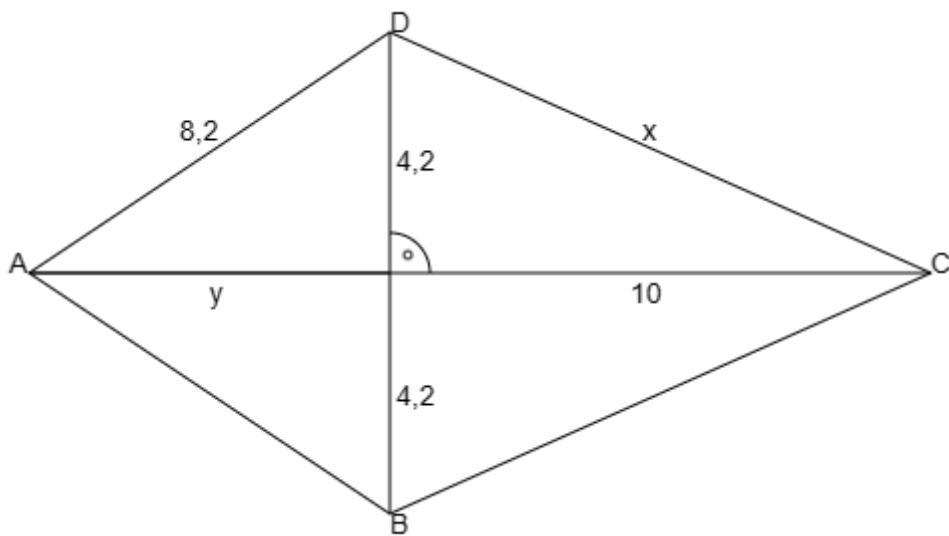
Nr. Grafik:



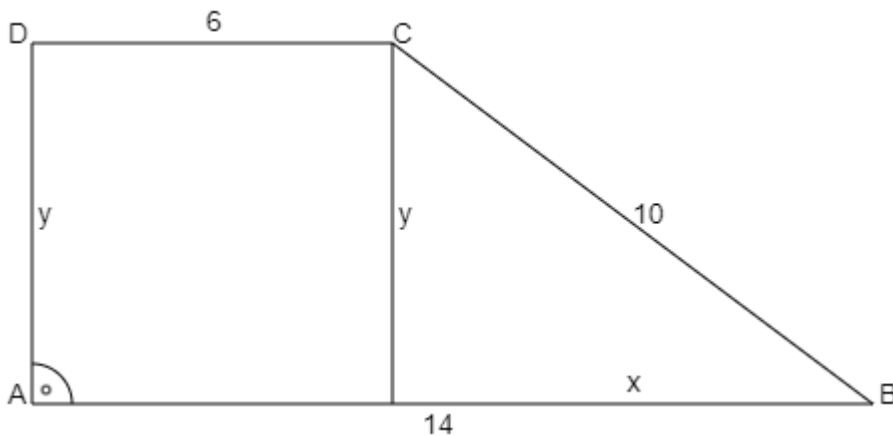
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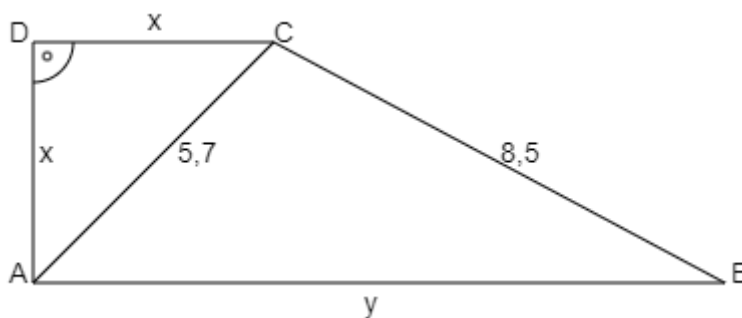
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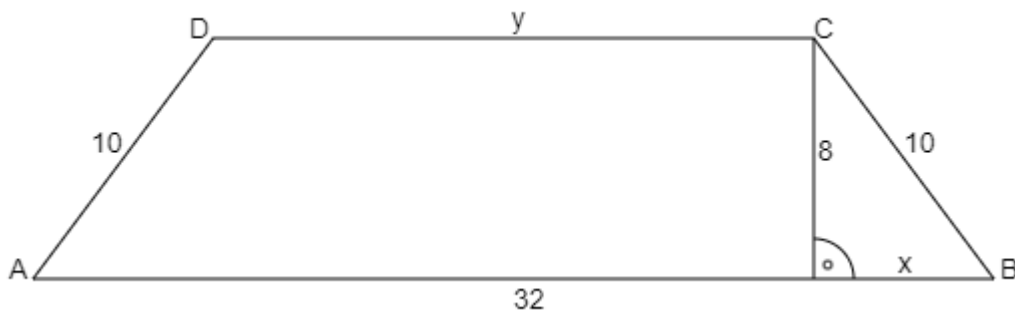
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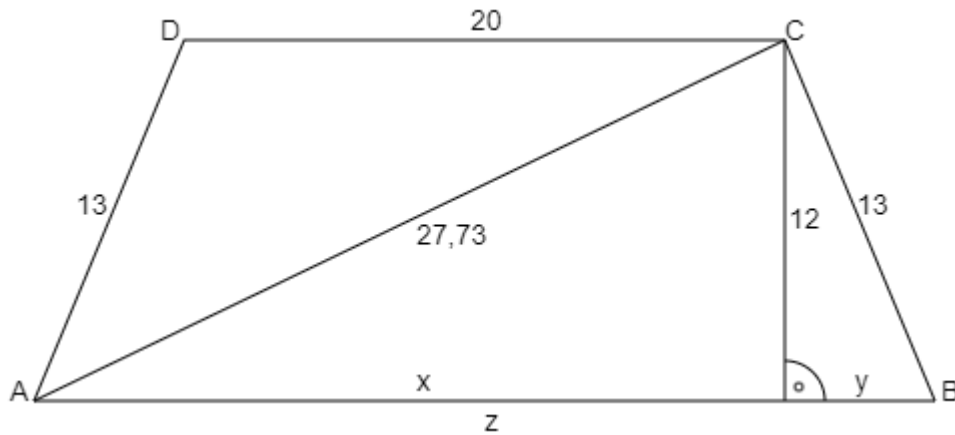
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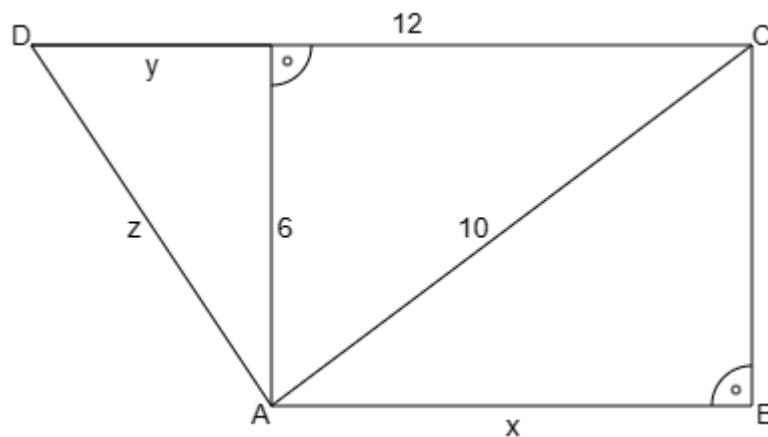
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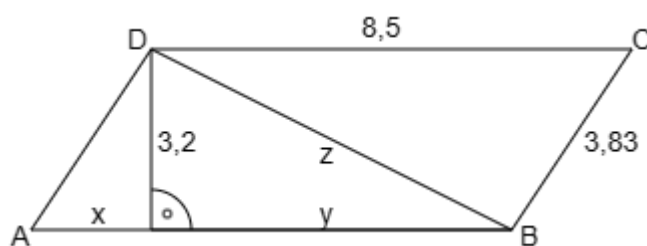
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**Vorgehensweise:** Zur Ermittlung der fehlenden Größen bei den Vierecken ist die obige Formelsammlung anzuwenden.

**Lösungen:** 1) Quadrat:  $x = 4,24$  cm,  $u = 16,97$  cm,  $A = 18$  cm<sup>2</sup>; 2) Rechteck:  $x = 4,5$  cm,  $u = 16$  cm,  $A = 15,75$  cm<sup>2</sup>; 3) Drache:  $x = 9,8$  cm,  $y = 5,7$  cm,  $u = 31$  cm,  $A = 52$  cm<sup>2</sup>; 4) Drache:  $x = 10,8$  cm,  $y = 7$  cm,  $u = 38$  cm,  $A = 71,4$  cm<sup>2</sup>; 5) Trapez:  $x = 8$  cm,  $y = 6$  cm,  $u = 36$  cm,  $A = 60$  cm<sup>2</sup>; 6) Trapez:  $x = 4$  cm,  $y = 11,5$  cm,  $u = 28$  cm,  $A = 31$  cm<sup>2</sup>; 7) Trapez:  $x = 6$  cm,  $y = 20$  cm,  $u = 72$  cm,  $A = 208$  cm<sup>2</sup>; 8) Trapez:  $x = 25$  cm,  $y = 5$  cm,  $z = 30$  cm,  $u = 76$  cm,  $A = 300$  cm<sup>2</sup>; 9) Trapez:  $x = 8$  cm,  $y = 4$  cm,  $z = 7,2$  cm,  $u = 33,2$  cm,  $A = 60$  cm<sup>2</sup>; 10) Parallelogramm:  $x = 2,1$  cm,  $y = 6,4$  cm,  $z = 7,2$  cm,  $u = 24,7$  cm,  $A = 27,2$  cm<sup>2</sup>.