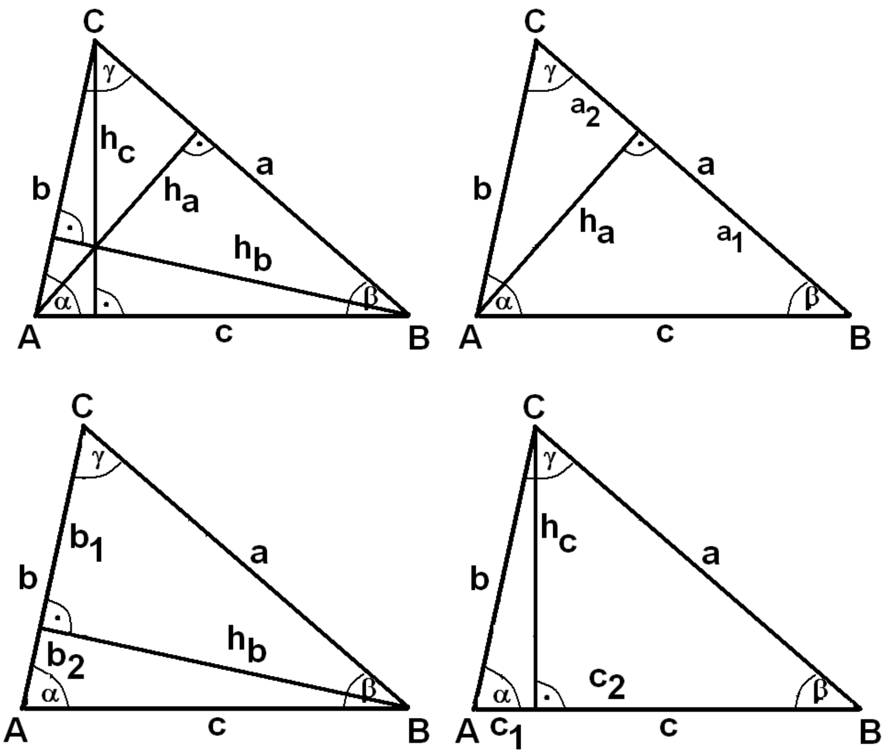
Michael Buhlmann

Mathematik-Formelsammlung
> Geometrie
> Trigonometrie (Dreiecksberechnung)
> Beliebige Dreiecke

Eine ebene geometrische Figur aus drei Punkten (Ecken) A, B, C und den <u>Seiten</u> *a, b, c* heißt <u>Dreieck</u> ΔABC, das Rechnen mit Dreiecken nennt man <u>Trigonometrie</u>. Die <u>Winkel</u> im Dreieck heißen α, β, γ und liegen bei den Punkten A, B, C. Gegeben sei ein beliebiges Dreieck ΔABC mit den Seiten a, b, c und den Winkeln α, β, γ. Der Inkreis berührt die Dreieckseiten, der Umkreis läuft durch die Dreiecksecken. Die drei Seitenhalbierenden halbieren jeweils von der gegenüberliegenden Ecke aus die Dreiecksseite, die drei Winkelhalbierenden halbieren die jeweiligen Dreieckswinkel.

Beliebige Dreiecke



Winkelsumme	$\alpha+\beta+\gamma = 180°$		
	$\alpha = 180° - \beta - \gamma$	$\beta = 180\text{“} - \alpha - \gamma$	$\gamma = 180° - \alpha - \beta$
Umfang	U = a + b + c		
	a = U – b – c	b = U – a – c	c = U – a – b
Flächeninhalt	$A = \frac{1}{2} a h_a$	$A = \frac{1}{2} b h_b$	$A = \frac{1}{2} c h_c$
	$a = \frac{2A}{h_a}$	$h_a = \frac{2A}{a}$	

	$b=\frac{2A}{h_b}$	$h_b=\frac{2A}{b}$	
	$c=\frac{2A}{h_c}$	$h_c=\frac{2A}{c}$	
	$A=\frac{1}{2}ab\sin\gamma$	$A=\frac{1}{2}ac\sin\beta$	$A=\frac{1}{2}bc\sin\alpha$
	$a=\frac{2A}{b\sin\gamma}$	$b=\frac{2A}{a\sin\gamma}$	$\sin\gamma=\frac{2A}{ab}$
	$a=\frac{2A}{c\sin\beta}$	$c=\frac{2A}{a\sin\beta}$	$\sin\beta=\frac{2A}{ac}$
	$b=\frac{2A}{c\sin\alpha}$	$c=\frac{2A}{b\sin\alpha}$	$\sin\alpha=\frac{2A}{bc}$
	$A=\frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$	$A=\frac{b^2\sin\alpha\sin\gamma}{2\sin\beta}$	$A=\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
$s=\frac{a+b+c}{2}$	$A=\sqrt{s(s-a)(s-b)(s-c)}$		(Heronsche Formel)
	$A=r_I^2\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$		
Höhen	$h_a=b\sin\gamma$	$h_b=a\sin\gamma$	$h_c=a\sin\beta$
	$h_a=c\sin\beta$	$h_b=c\sin\alpha$	$h_c=b\sin\alpha$
	$h_a:h_b:h_c=\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$		
	$\frac{h_a}{h_b}=\frac{b}{a}$	$\frac{h_a}{h_c}=\frac{c}{a}$	$\frac{h_b}{h_c}=\frac{c}{b}$
Satz des Pythagoras	$a_1^2+h_a^2=c^2$ $c=\sqrt{a_1^2+h_a^2}$	$a_2^2+h_a^2=b^2$ $b=\sqrt{a_2^2+h_a^2}$	$h_a^2=c^2-a_1^2$ $h_a=\sqrt{c^2-a_1^2}$
$a_1 + a_2 = a$ bzw. $a_1 - a_2 = a$ bzw. $a_2 - a_1 = a$	$a_1^2=c^2-h_a^2$ $a_1=\sqrt{c^2-h_a^2}$	$a_2^2=b^2-h_a^2$ $a_2=\sqrt{b^2-h_a^2}$	$h_a^2=b^2-a_2^2$ $h_a=\sqrt{b^2-a_2^2}$
	$b_1^2+h_b^2=a^2$ $a=\sqrt{b_1^2+h_b^2}$	$b_2^2+h_b^2=c^2$ $c=\sqrt{b_2^2+h_b^2}$	$h_b^2=a^2-b_1^2$ $h_b=\sqrt{a^2-b_1^2}$
$b_1 + b_2 = b$ bzw. $b_1 - b_2 = b$ bzw. $b_2 - b_1 = b$	$b_1^2=a^2-h_b^2$ $b_1=\sqrt{a^2-h_b^2}$	$b_2^2=c^2-h_b^2$ $b_2=\sqrt{c^2-h_b^2}$	$h_b^2=c^2-b_2^2$ $h_b=\sqrt{c^2-b_2^2}$

	$c_1^2 + h_c^2 = b^2$ $b = \sqrt{c_1^2 + h_c^2}$	$c_2^2 + h_c^2 = a^2$ $a = \sqrt{c_2^2 + h_c^2}$	$h_c^2 = b^2 - c_1^2$ $h_c = \sqrt{b^2 - c_1^2}$
$c_1 + c_2 = c$ bzw. $c_1 - c_2 = c$ bzw. $c_2 - c_1 = c$	$c_1^2 = b^2 - h_c^2$ $c_1 = \sqrt{b^2 - h_c^2}$	$c_2^2 = a^2 - h_c^2$ $c_2 = \sqrt{a^2 - h_c^2}$	$h_c^2 = a^2 - c_2^2$ $h_c = \sqrt{a^2 - c_2^2}$
Trigonometrische Funktionen	$\sin\alpha = \frac{h_c}{b}$	$b = \frac{h_c}{\sin\alpha}$	$h_c = b\sin\alpha$
	$\cos\alpha = \frac{c_1}{b}$	$b = \frac{c_1}{\cos\alpha}$	$c_1 = b\cos\alpha$
	$\tan\alpha = \frac{h_c}{c_1}$	$c_1 = \frac{h_c}{\tan\alpha}$	$h_c = c_1\tan\alpha$
	$\sin\alpha = \frac{h_b}{c}$	$c = \frac{h_b}{\sin\alpha}$	$h_b = c\sin\alpha$
	$\cos\alpha = \frac{b_2}{c}$	$c = \frac{b_2}{\cos\alpha}$	$b_2 = c\cos\alpha$
	$\tan\alpha = \frac{h_b}{b_2}$	$b_2 = \frac{h_b}{\tan\alpha}$	$h_b = b_2\tan\alpha$
	$\sin\beta = \frac{h_c}{a}$	$a = \frac{h_c}{\sin\beta}$	$h_c = a\sin\beta$
	$\cos\beta = \frac{c_2}{a}$	$a = \frac{c_2}{\cos\beta}$	$c_2 = a\cos\beta$
	$\tan\beta = \frac{h_c}{c_2}$	$c_2 = \frac{h_c}{\tan\beta}$	$h_c = c_2\tan\beta$
	$\sin\beta = \frac{h_a}{c}$	$c = \frac{h_a}{\sin\beta}$	$h_a = c\sin\beta$
	$\cos\beta = \frac{a_1}{c}$	$c = \frac{a_1}{\cos\beta}$	$a_1 = c\cos\beta$
	$\tan\beta = \frac{h_a}{a_1}$	$a_1 = \frac{h_a}{\tan\beta}$	$h_a = a_1\tan\beta$
	$\sin\gamma = \frac{h_b}{a}$	$a = \frac{h_b}{\sin\gamma}$	$h_b = a\sin\gamma$
	$\cos\gamma = \frac{b_1}{a}$	$a = \frac{b_1}{\cos\gamma}$	$b_1 = a\cos\gamma$
	$\tan\gamma = \frac{h_b}{b_1}$	$b_1 = \frac{h_b}{\tan\beta}$	$h_b = b_1\tan\beta$

	$\sin\gamma = \frac{h_a}{b}$	$b = \frac{h_a}{\sin\gamma}$	$h_a = b\sin\gamma$
	$\cos\gamma = \frac{a_2}{b}$	$b = \frac{a_2}{\cos\gamma}$	$a_2 = b\cos\gamma$
	$\tan\gamma = \frac{h_a}{a_2}$	$a_2 = \frac{h_a}{\tan\gamma}$	$h_a = a_2 \tan\gamma$
Sinussatz	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$		
	$\frac{a}{b} = \frac{\sin\alpha}{\sin\beta}$	$\frac{b}{c} = \frac{\sin\beta}{\sin\gamma}$	$\frac{a}{c} = \frac{\sin\alpha}{\sin\gamma}$
$a = \frac{\sin\alpha}{\sin\beta} b$	$b = \frac{\sin\beta}{\sin\alpha} a$	$\sin\alpha = \frac{a}{b}\sin\beta$	$\sin\beta = \frac{b}{a}\sin\alpha$
$b = \frac{\sin\beta}{\sin\gamma} c$	$c = \frac{\sin\gamma}{\sin\beta} b$	$\sin\beta = \frac{b}{c}\sin\gamma$	$\sin\gamma = \frac{c}{b}\sin\beta$
$a = \frac{\sin\alpha}{\sin\gamma} c$	$c = \frac{\sin\gamma}{\sin\alpha} a$	$\sin\alpha = \frac{a}{c}\sin\gamma$	$\sin\gamma = \frac{c}{a}\sin\alpha$
Cosinussatz	$a^2 = b^2 + c^2 - 2bc\cos\alpha$		$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$
	$b^2 = c^2 + a^2 - 2ca\cos\beta$		$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$
	$c^2 = a^2 + b^2 - 2ab\cos\gamma$		$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$
Mollweidesche Formeln	$\frac{a+b}{c} = \frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}}$	$\frac{a-b}{c} = \frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\gamma}{2}}$	
	$\frac{b+c}{a} = \frac{\cos\frac{\beta-\gamma}{2}}{\sin\frac{\alpha}{2}}$	$\frac{b-c}{a} = \frac{\sin\frac{\beta-\gamma}{2}}{\cos\frac{\alpha}{2}}$	
	$\frac{c+a}{b} = \frac{\cos\frac{\gamma-\alpha}{2}}{\sin\frac{\beta}{2}}$	$\frac{c-a}{b} = \frac{\sin\frac{\gamma-\alpha}{2}}{\cos\frac{\beta}{2}}$	
Tangenssatz	$\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}} = \frac{\cot\frac{\gamma}{2}}{\tan\frac{\alpha-\beta}{2}}$		

	$\frac{b+c}{b-c}=\frac{\tan\frac{\beta+\gamma}{2}}{\tan\frac{\beta-\gamma}{2}}=\frac{\cot\frac{\alpha}{2}}{\tan\frac{\beta-\gamma}{2}}$		
	$\frac{a+c}{a-c}=\frac{\tan\frac{\alpha+\gamma}{2}}{\tan\frac{\alpha-\gamma}{2}}=\frac{\cot\frac{\beta}{2}}{\tan\frac{\alpha-\gamma}{2}}$		
Halbwinkelsätze	$\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$	$\sin\frac{\beta}{2}=\sqrt{\frac{(s-a)(s-c)}{ac}}$	$\sin\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$
$s=\frac{a+b+c}{2}$	$\cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}}$	$\cos\frac{\beta}{2}=\sqrt{\frac{s(s-b)}{ac}}$	$\cos\frac{\gamma}{2}=\sqrt{\frac{s(s-c)}{ab}}$
	$\tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$	$\tan\frac{\beta}{2}=\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$	$\tan\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$
Seitensätze	a + b > c	b + c > a	a + c > b
Seitenhalbierende	$s_a=\frac{1}{2}\sqrt{2(b^2+c^2)-a^2}=\frac{1}{2}\sqrt{b^2+c^2+2bc\cos\alpha}$		
	$s_b=\frac{1}{2}\sqrt{2(a^2+c^2)-b^2}=\frac{1}{2}\sqrt{a^2+c^2+2ac\cos\beta}$		
	$s_c=\frac{1}{2}\sqrt{2(a^2+b^2)-c^2}=\frac{1}{2}\sqrt{a^2+b^2+2ab\cos\gamma}$		
Winkelhalbierende	$w_\alpha=\frac{1}{b+c}\sqrt{bc[(b+c)^2-a^2]}=\frac{2bc\cos\frac{\alpha}{2}}{b+c}$		
	$w_\beta=\frac{1}{a+c}\sqrt{ac[(a+c)^2-b^2]}=\frac{2ac\cos\frac{\beta}{2}}{a+c}$		
	$w_\gamma=\frac{1}{a+b}\sqrt{ab[(a+b)^2-c^2]}=\frac{2ab\cos\frac{\gamma}{2}}{a+b}$		
Umkreisradius	$r_U=\frac{a}{2\sin\alpha}=\frac{b}{2\sin\beta}=\frac{c}{2\sin\gamma}$		
	$r_U=\frac{bc}{2h_a}=\frac{ac}{2h_b}=\frac{ab}{2h_c}$		
Inkreisradius	$r_I=\frac{A}{s}=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$		
$s=\frac{a+b+c}{2}$	$r_I=(s-a)\tan\frac{\alpha}{2}$	$r_I=(s-b)\tan\frac{\beta}{2}$	$r_I=(s-c)\tan\frac{\gamma}{2}$

	$r_I = s\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$	
	$r_I = 4r_U \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$	
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